

New results on the Lambert W function

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A number of new results on the Lambert W function [1] and applications are presented. The asymptotic behaviour of the non-principal branches $W_k(z)$, $k \neq 0$ around $z = 0$ has been overlooked until now; the expansions are now detailed. The asymptotic expansions at $|z| \rightarrow \infty$ are known to be convergent for large z . It is now possible to give the boundary in the complex plane of the convergence domain. Various expressions containing W are Stieltjes functions [2]; direct proofs are given. Also, there are applications in statistics of the $k = -1$ branch, and these applications lead to Stieltjes functions there also. Recently, a proposal was presented regarding alternative branch structures for W [3]. This proposal will be discussed.

References

- [1] Robert M Corless, Gaston H Gonnet, DEG Hare, David J Jeffrey, Donald E Knuth, *On the Lambert W function*, Advances in Computational Mathematics, Vol 5 (1996) pp 329–359.
- [2] German A Kalugin, David J Jeffrey, Robert M Corless, Peter B Borwein, *Stieltjes and other integral representations for functions of Lambert W* , Integral Transforms and Special Functions, (2012), pp. 581–593.
- [3] D. J. Jeffrey, Tree T and Lambert W , (2021) DOI: 10.1109/SYNASC54541.2021.00015