Generalized hypergeometric solutions of the Fuchsian linear differential equations

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We present infinitely many solutions of the general Heun equation in terms of the generalized hypergeometric functions ${}_{p+1}F_p$. Each solution assumes two restrictions imposed on the involved parameters: a characteristic exponent of a singularity should be a non-zero integer and the accessory parameter should obey a polynomial equation. [1],[2]

Next, we show that the single confluent Heun equation with non-zero ε (this is the parameter characterizing the irregular singularity at the infinity) admits infinitely many solutions in terms of the generalized hypergeometric functions ${}_pF_p$. For each of these solutions a characteristic exponent of a regular singularity of the confluent Heun equation is a non-zero integer and the accessory parameter obeys a polynomial equation. Each solution can be written as a linear combination with constant coefficients of a finite number of the Kummer confluent hypergeometric functions. [3]

Furthermore, we show that for the Ince limit $\varepsilon = 0$ the confluent Heun equation admits infinitely many solutions in terms of the functions ${}_pF_{p+1}$. Here again a characteristic exponent of a regular singularity should be a non-zero integer and the accessory parameter should obey a polynomial equation. This time, each solution can be written as a linear combination with constant coefficients of a finite number of the Bessel functions. [3]

Finally, we show that a Fuchsian differential equation having five regular singular points admits solutions in terms of a single generalized hypergeometric function for infinitely many particular choices of equation parameters. Each solution assumes four restrictions imposed on the parameters: two of the singularities should have non-zero integer characteristic exponents and the accessory parameters should obey polynomial equations. [4]

References

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