Computation of confluent hypergeometric functions and applications

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Confluent hypergeometric functions occur in many applications in applied mathematics, physics and engineering. Despite their importance, few algorithms are available for calculating any of the standard solutions of Kummer's equation in the case of real or complex parameters. In this talk, we present recent advances in the computation of the Kummer function U(a, b, x) [1]. As we will see, asymptotic expansions [2] are very important in the resulting algorithm. On the other hand, confluent hypergeometric functions play a key role in the asymptotic analysis of Fermi-Dirac integrals [3]. The evaluation of these integrals and their derivatives is necessary for various problems in applied and theoretical physics, such as stellar astrophysics, plasma physics or electronics. In this lecture we will show that the use of these expansions makes it possible to calculate the functions efficiently and with high accuracy for a large number of parameters [4].

References

[1] A. Gil, D. Ruiz-Antolín, J. Segura, N.M. Temme. *Efficient and accurate computation of confluent hypergeometric function $U(a, b, x)$*. In progress.

[2] N.M. Temme. *Asymptotic expansions of Kummer hypergeometric functions for large values of the parameters*. Integral Transforms Spec. Funct. 33(1) (2022) 16-31.

[3] A. Gil, J. Segura, N.M. Temme. *Complete asymptotic expansions for the relativistic Fermi-Dirac integral*. Appl. Math. Comput. 412 (2022) 126618. https://arxiv.org/abs/2108.11210v2

[4] A. Gil, A. Odrzywolek, J. Segura, N.M. Temme. *Evaluation of the relativistic Fermi-Dirac integral and its derivatives for moderate/large values of the parameters*. In progress.

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