A class of special functions using Fourier transforms of orthogonal polynomials on the unit disk

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The study of orthogonal polynomials and their transformations have been the subject of many papers during the last several years. By the Fourier transform or other integral transforms, some univariate orthogonal polynomials systems which are mapped onto each other have been studied in [5, 7]. Recently, there have been many papers on Fourier transforms of univariate and multivariate orthogonal polynomials [3, 4, 6, 8].

In this work, we present Fourier transform of multivariate orthogonal polynomials on the unit disk $B^r = \{ \boldsymbol{x} \in \mathbb{R}^r : \|\boldsymbol{x}\| \leq 1 \}$ (see [2]) and we write them in terms of continuous Hahn polynomials. By using the obtained Fourier transforms and Parseval's identity [1], we derive a new family of multivariate orthogonal functions.

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