Central part interpolation schemes for fractional differential equations

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A class of initial value problems for fractional integro-differential equations involving a Caputo fractional differential operator of order $\alpha \in (0, 1)$ is considered. First, the problem is reformulated as a weakly singular Volterra integral equation of the second kind. Then, a smoothing change of variables is used to improve the boundary behaviour of the exact solution of the underlying problem. After that, a collocation method based on central part interpolation by continuous piecewise polynomials on the uniform grid is constructed. The central part interpolation approach was introduced in [1] for solving Fredholm integral equations of the second kind and it has shown accuracy and numerical stability advantages compared to standard piecewise polynomial collocation methods, including collocation at Chebyshev knots [2]. In the present talk this approach is modified to solve fractional differential equations. The optimal convergence estimates are derived and the theoretical results are tested by some numerical experiments.

References

- K. Orav-Puurand, G. Vainikko, Central part interpolation schemes for integral equations, Numer. Func. Anal. Optim., 30 (2009), pp. 352–370.
- [2] K. Orav-Puurand, A. Pedas, G. Vainikko, Central part interpolation schemes for integral equations with singularities, J. Integral Equations Appl., 29 (2017), pp. 401–440.