On the convergence of difference approximations to fractional differential problems in bounded domains

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Fractional differential equations are promising tools for characterizing anomalous diffusion in different fields and many interesting problems occur in bounded domains [1, 3]. When we add physical boundaries to fractional differential equations many difficulties arise and these include the development of accurate numerical approximations. Because the fractional derivative is nonlocal, the accuracy of its approximation near the boundary is strongly affected by the cut of the domain, that is, in general, the accuracy of the approximations of fractional order operators is lost near the boundary. To discretize these equations and in particular the fractional derivative we consider a known approximation that can be first order accurate when we have an open domain but it can be of lower order in bounded domains and sometimes not consistent [2]. Nevertheless, as we will show, the numerical methods are convergent and the first order rate of convergence can be recovered.

References

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