A general collocation analysis for weakly singular Volterra integral equations with variable exponent

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Piecewise polynomial collocation of weakly singular Volterra integral equations (VIEs) of the second kind has been extensively studied in the literature, where integral kernels of the form $(t-s)^{-\alpha}$ for some constant $\alpha \in (0,1)$ are considered. Variable-order fractional-derivative differential equations currently attract much research interest, and in Zheng and Wang SIAM J. Numer. Anal. 2020 such a problem is transformed to a weakly singular VIE whose kernel has the above form with variable $\alpha = \alpha(t)$, then solved numerically by piecewise linear collocation, but it is unclear whether this analysis could be extended to more general problems or to polynomials of higher degree. In the present paper the general theory (existence, uniqueness, regularity of solutions) of variable-exponent weakly singular VIEs is developed, then used to underpin an analysis of collocation methods where piecewise polynomials of any degree can be used. The sharpness of the theoretical error bounds obtained for the collocation methods is demonstrated by numerical examples.

References

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