Real exponential asymptotic behaviour is generic in the mean square of two-dimensional linear SDE's

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In this talk we are concerned with the mean square behaviour of linear 2-dimensional systems of stochastic differential equations with constant coefficients. The behaviour of such systems has been studied in great detail over the past fifty years and in particular a full characterisation of the mean square of n-dimensional systems of SDE's was given by Ludwig Arnold. Using Arnold's approach, one can show the mean square process obeys an $n^2 \times n^2$ system of ordinary differential equations, hence all information regarding the mean square process can be inferred by studying the eigenstructure of the associated $n^2 \times n^2$ coefficient matrix. For high dimensional problems finding exact evolutionary behaviour becomes highly intractable but for the case where $n = 2$, such computational issues can be surmounted and it is such eigenvalue analysis to which this talk is devoted. In particular we show that for arbitrarily small and large noise the dominant dynamics are **always** real exponential where dominance is classified by the long term behaviour of solutions of the mean square process. We also identify special cases wherein real exponential behaviour prevails independent of the level of noise introduced into the system. Our approach is to make a suitable coordinate transformation which results in a reduction of complexity in the characteristic polynomial of the associated 4×4 matrix. It should be noted we make no prior assumptions on structure of the underlying coefficient matrices to allow for a completely general treatment of the mean square. It is of particular interest that upon introduction of arbitrarily small noise into the underlying deterministic system, solutions are **unable** to produce oscillatory behaviour, even in the case when the solution of the noise-free equation has oscillatory solutions. The remarkable rarity of dominant oscillatory solutions, regardless of the eigenstructure of the drift and diffusion matrices in the two-dimensional case, leads to the conjecture that the phenomenon of dominant real exponential behaviour in the mean square may extend to arbitrarily many dimensions. However, the methods of proof used in this talk would not readily generalise to attack this conjecture. These are preliminary results in part of a larger study with Conall Kelly (UCC) on the seemingly generic appearance of dominant real exponential asymptotic behaviour in the mean square in autonomous linear stochastic equations.