Numerical schemes for a class of singular fractional integro-differential equations

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Fractional derivatives and equations containing them have fascinated scientists for a very long time. Over the last few decades interest in the field has increased significantly because of new applications in Physics, Chemistry, Electrical Networks and so on [1, 3].

In [2], the unique solvability of singular fractional differential equations was studied. In the current talk we consider singular fractional integro-differential equations of the form

$$(D_0^\alpha M^\alpha u)(t) = \sum_{k=1}^{l} b_k (D_0^{\alpha_k} M^{\alpha_k} u)(t) + b(Vu)(t) + f(t), \quad 0 < t \le T, \tag{1}$$

where the multiplication operator M^ν is defined by

$$(M^\nu u)(t) = t^\nu u(t), \quad 0 < t \le T, \quad \nu \in \mathbb{R}, \quad u \in C[0,T],$$

V is a certain type of Volterra integral operator, $\alpha, \alpha_k, b, b_k, \in \mathbb{R}$, and

$$m < \alpha \le m+1, \quad \alpha > \alpha_k \ge 0, \quad f \in C^m[0,T], \quad k = 1,2,\dots,l, \quad m \in \mathbb{N}_0 = \{0,1,2,\dots\}.$$

By $C^m[0,T]$ ($m \in \mathbb{N}_0$) we denote the space of m times continuously differentiable functions u on $[0,T]$; $C^0[0,T] = C[0,T]$. In equation (1) the fractional differential operator D_0^μ, of order $\mu \in [0,\infty)$, is defined as the inverse of the Riemann-Liouville integral operator

$$(J^\mu u)(t) = \frac{1}{\Gamma(\mu)} \int_0^t (t-s)^{\mu-1} u(s)ds, \quad u \in C[0,T], \quad t > 0, \ \mu > 0; \quad J^0 = I,$$

where I is the identity mapping and Γ the Euler gamma function.

In the talk we present some results about the unique solvability of equations of the form (1) and introduce a collocation based scheme for finding the numerical solution of such equations. We also give results of numerical experiments.

References

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