A posteriori error estimates for time discretization of abstract semi–linear fractional integro–differential equations

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The aim of this work is to provide a posteriori error estimates for time discretization of abstract semi–linear time fractional equations

$$\partial_t^\beta u(t) = Au(t) + F(u(t)), \qquad 0 < t \leq T, \tag{1}$$

where ∂_t^β stands for the time fractional derivative operator of order $\beta > 0$ in Riemann–Liouville sense, $1 < \beta < 2$, A is an abstract linear operator in a complex Banach space X, $A : D(A) \subset X \to X$, and $F(u)$ a reaction term under certain regularity conditions.
Our a posteriori error estimates (see [2]) are achieved through the maximal Hölder regularity of the analytical solution $u(t)$ to (1) in the context of θ–sectorial operators A, $0 < \theta < \pi/2$. This approach has been previously considered for abstract ordinary differential equations, that is those where first time derivative is considered instead of fractional ones ∂_t^β (see [1]). In the fractional case the main difficulty arises from the lack of regularity typically occurs for solutions to differential equations involving time fractional derivatives and/or integrals. Even though our work focused on providing estimates in a theoretical framework, throughout the work we show that all constants involved in the final estimates are in actual fact computables in the spirit of genuine a posteriori error esrtimates.

References

[1] E. Cuesta, Ch. Makridakis, *A posteriori error estimates and maximal regularity for approximations of fully nonlinear parabolic problems in Banach spaces*, Numer. Math. 111 (3) (2008), pp. 257–275.

[2] E. Cuesta, R. Ponce, *Hölder regularity for abstract semi–linear fractional differential equations in Banach spaces*, Comput. Math. Appl., 85 (2021), pp. 57–68.