Highly accurate solution of fractional differential equations

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This talk deals with the numerical solution of nonlinear fractional differential equations (FDEs) of type

$$\begin{cases} D^{\alpha}y(t) = f(t, y(t)), & 0 \le t \le b, \\ y^{(i)}(0) = \gamma_i, & i = 0, \dots, n-1, \end{cases}$$

where $n-1 < \alpha < n$, $n \in \mathbb{N}$, $\gamma_i \in \mathbb{R}$, $f : [0,b] \times \mathbb{R} \to \mathbb{R}$ is a given continuous function. The fractional derivative $D^{\alpha}y$ is the Caputo-type one [5]. FDEs arise in various fields, such as the dynamics in viscoelastic materials, the evolution of certain diseases, especially when the modelled phenomenon heavily depends on its past history.

On the side of the numerical simulation, solving FDEs with high accuracy is a challenging issue, since many numerical methods have low order of convergence. A powerful technique to obtain high order methods without increasing the computational cost consists of multistep collocation. We propose the class of two-step spline collocation methods [1, 2, 3], which double the order of convergence of the one-step collocation methods [4], at the same computational cost. In this talk, we analyze the convergence and stability properties of these methods, illustrate the main issues related to the implementation and finally show some numerical experiments.

References

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