On the computation of the Wright function and its applications to Fractional Calculus

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The Wright function is defined by the following power series, convergent in the whole complex plane,

$$W_{\lambda,\mu}(z) := \sum_{n=0}^{\infty} \frac{z^n}{n!\,\Gamma(\lambda n + \mu)}, \quad \lambda > -1,\, \mu \in \mathbb{C}. \tag{1}$$

Originally Wright assumed $\lambda \geq 0$ in connection with his investigation of the asymptotic theory of partition [6] and only in 1940 he considered $\lambda \in (-1, 0)$. The latter case is now referred to in the literature as *Wright function of the second kind* (WF2K) [7]. Although several representations of the Wright function have been introduced and many of its analytical properties have already been well studied (see, e.g., [2, 3, 4, 5]), its numerical computation is still an active research area.
In this talk we devote our attention to the numerical evaluation of WF2K, since this is the most interesting case for applications. We approach this topic by considering a technique based on the numerical inversion of the Laplace transform combined with a trapezoidal rule on a parabolic contour. We present some numerical experiments that validate both the theoretical estimates of the error and the applicability of the proposed technique to represent the solutions of fractional differential equations [1]. A code package that implements the algorithm proposed is contained in the repository: `github.com/Cirdans-Home/mwright`.

References

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