Iterative Shepard operator of least squares thin-plate spline type

Andra Malina[a]

[a] Department of Mathematics, Babeș-Bolyai University Cluj-Napoca (Romania)
andra.malina@ubbcluj.ro

D. Shepard introduced in 1968 in [5] a very powerful method for approximating a given function f on a set of scattered data. A method for improving its accuracy was introduced by R. Franke and G. Nielson in [2].
T. Cătinaș and A. Malina introduced in [1] a new Shepard operator, based on the classical and the modified Shepard methods and the least-squares thin-plate spline function. The least squares thin-plate spline is defined for a point (x, y) and a set of nodes (x_i, y_i) as

$$F_i(x,y) = \sum_{j=1}^{i} C_j d_j^2 \log(d_j) + ax + by + c,\ i = 1, ..., N'$$

with $d_j = \sqrt{(x-x_j)^2 + (y-y_j)^2}$ and $C_j,\ a,\ b,\ c$ found such that they minimize

$$E = \sum_{i=1}^{N'} [F_i(x_i, y_i) - f(x_i, y_i)]^2,$$

for two different values of N', first with $N' = N$ interpolation nodes and second with $N' = k$ representative knot points, idea presented by J. McMahon in [4].
We propose an iterative modification of the Shepard operator of least squares thin-plate spline type, following an idea presented by A. Masjukov and V. Masjukov in [3]. The operator, denoted by u_L , is defined as

$$u_L(x,y) = \sum_{k=0}^{K} \sum_{j=1}^{N'} \left[u_{F_j}^{(k)} w\left((x-x_j, y-y_j)/\tau_k\right) / \sum_{p=1}^{N'} w\left((x_p - x_j, y_p - y_j)/\tau_k\right) \right],$$

where w is a continuously differentiable weight function with some particular properties, $u_{F_j}^{(k)}$ denotes the interpolation residuals at the kth step and τ_k is a scaling parameter.

References

[1] T. Cătinaş, A. Malina, *Shepard operator of least squares thin-plate spline type*, Stud. Univ. Babeş-Bolyai Math. **66** (2021), pp. 257–265.

[2] R. Franke, G. Nielson, *Smooth interpolation of large sets of scattered data.* Int. J. Numer. Meths. Engrg. **15** (1980), pp. 1691–1704.

[3] A. Masjukov, V. Masjukov, *Multiscale modification of Shepard's method for interpolation of multivariate scattered data*, in: Mathematical Modelling and Analysis, Proc. 10th Internat. Conf. MMA & CMAM2, 2005, pp. 467–472.

[4] J. McMahon,*Knot selection for least squares approximation using thin plate splines*, M.S. Thesis, Naval Postgraduate School, 1986.

[5] D. Shepard, *A two dimensional interpolation function for irregularly spaced data*, in: Proc. 23rd Nat. Conf. ACM, 1968, pp. 517–523.