# The weighted Weierstrass Theorem for continuous functions defined on $[0, \infty)$ or on $(-\infty, \infty)$, proved using Bernstein-Chlodovski operators

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The Chlodovski extensions [3] of the classical Bernstein operators [2] were used in Kilgore [4] and [5] to prove the weighted versions of the classical Weierstrass Approximation Theorem, in the situation that the functions to be approximated are defined and continuous upon the interval $[0, \infty)$ using the weight $W(x) = e^{-x^\alpha}$ and satisfy $W(X)f(x) \to 0$ as $x \to \infty$. And in the similar case that interval is $(-\infty, \infty)$ using the weight $W(x)e^{-|x|^\alpha}$, and $W(x)f(x) \to 0$ as $|x| \to \infty$.
In each of the two above-described contexts, the Weierstrass theorem was already known to hold, but the new proofs were simple, basic in character, completely self-contained and autonomous. However, to approximate continuous functions defined upon $[0, \infty)$ it was necessary in constructing the new proof to assume that $\alpha > 1$, and for continuous functions defined on $(-\infty, \infty)$ one needed to assume that $\alpha > 2$.
Here, it is shown in each case above that the admissible value of $\alpha$ can be reduced. For the approximation on $[0, \infty)$ one may assume that $\alpha > \frac{1}{2}$. And the approximation on $(-\infty, \infty)$ requires $\alpha > 1$. As is known from [1] or [6], these are the least possible values of $\alpha$ for which the weighted version of the Weierstrass approximation can hold in the two respective situations. The proofs of these new results follow from minor changes to the Chlodovski operators.

## References

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