Voronovskaja type results for the Aldaz, Kounchev, Render modification of Baskakov type operators

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For $j > 1, j \in \mathbb{N}$ fixed and $n \ge j$, Aldaz, Kounchev and Render [1] introduced a modification of the Bernstein operator that preserves the monomials e_0 and e_j . This operator $B_{n,j}: C[0,1] \to C[0,1]$ is constructed using the classical original Bernstein basis functions $p_{n,k}(x) = \binom{n}{k} x^k (1-x)^{n-k}, x \in [0,1]$ and is explicitly given by

$$B_{n,j}(f;x) = \sum_{k=0}^{n} f\left(t_{n,k}^{(j)}\right) p_{n,k}(x), \qquad (1)$$

where

$$t_{n,k}^{(j)} = \left(\frac{k(k-1)\dots(k-j+1)}{n(n-1)\dots(n-j+1)}\right)^{1/j}$$

We generalize the definition to Baskakov type operators and prove a corresponding Voronovskaja type result.

References

 J.M. Aldaz, O. Kounchev, H. Render, Shape preserving properties of generalized Bernstein operators on extended Chebyshev spaces, Numer. Math., 2009, 114(1), 1–25