Metric Fourier approximation of set-valued functions of bounded variation

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We study set-valued functions (SVFs) mapping a real interval to compact sets in $\mathbb{R}^d$. Older approaches to the approximation of set-valued functions investigated almost exclusively SVFs with convex images (values). The standard methods suffer from convexification.
In this talk I will describe a new construction that adopts the trigonometric Fourier series to set-valued functions with general (not necessarily convex) compact images. Our main result is analogous to the classical Dirichlet-Jordan Theorem for real functions. It states the pointwise convergence in the Hausdorff metric of the metric Fourier partial sums of a set-valued function of bounded variation to a set determined by the values of the metric selections of the function. In particular, if the set-valued F is of bounded variation and continuous at a point x, then the metric Fourier partial sums of it at x converge to $F(x)$. If F is continuous in a closed interval, then the convergence is uniform.