

On Wachnicki operators

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The talk aims at a generalization W_α ($\alpha \geq -1/2$) of the Gauss-Weierstrass integral operator introduced by Eugeniusz Wachnicki [1]. The operator is intimately connected to a generalization of the heat equation.

It is defined as follows

$$W_\alpha(f; r, t) = \frac{1}{2t} \int_0^\infty r^{-\alpha} s^{\alpha+1} \exp\left(-\frac{r^2 + s^2}{4t}\right) I_\alpha\left(\frac{rs}{2t}\right) f(s) ds,$$

where $(r, t) \in (0, \infty) \times (0, \infty)$ and I_α is the modified Bessel function of the first kind and fractional order α .

For $\alpha = -1/2$ the operator becomes the authentic Gauss-Weierstrass operator.

Our results focus on the asymptotic expansion of both W_α operators and their derivatives

$\left(\frac{\partial}{\partial r}\right)^m W_\alpha$ of any order $m \in \mathbb{N}$.

References

- [1] E. Wachnicki, *On a Gauss-Weierstrass generalized integral*, Rocznik Naukowo-Dydaktyczny Akademii Pedagogicznej W Krakowie, Prace Matematyczne, **17**(2000), pp. 251–263.