Regularized minimal-norm solution of overdetermined first kind integral models

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First kind integral equations arise in many mathematical models. A typical situation is when one needs to identify certain parameters of a physical system confined in a specified domain, from observations detected outside the domain by, e.g., mechanical or electromagnetic waves. Some sensing devices used in such settings allow different configurations, leading to overdetermined systems of integral equation with discrete data of the form

$$\int_a^b k_\ell(x_{\ell,i}, t)\, f(t)\, dt = g_\ell(x_{\ell,i}), \qquad \ell = 1, \ldots, m, \quad i = 1, \ldots, n_\ell,$$

Such problems are typically ill-posed: they admit infinitely many solutions and, because of experimental errors, must be solved in the least-squares sense.

We will describe a numerical method for computing the minimal-norm solution of the problem in the presence of boundary constraints, which stems from the Riesz representation theorem and the theory of reproducing kernel Hilbert spaces (RKHS) [1, 2]. The algorithm involves the singular system of the integral operator associated to the overdetermined system and naturally induces a regularization technique. Numerical experiments, both synthetic and deriving from an application in applied geophysics, show that the new method is extremely effective when the sought solution is smooth, but produces significant information even for non-smooth solutions.

References

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