Sinc-collocation methods with consistent collocation points for Fredholm integral equations of the second kind

Tomoaki Okayama[a]

[a] Graduate School of Information Sciences, Hiroshima City University (Japan)
okayama@hiroshima-cu.ac.jp

This talk is concerned with Fredholm integral equations of the second kind of the form

$$u(t) - \int_a^b k(t,s)u(s)\mathrm{d}s = g(t), \quad a \leq t \leq b,$$

where k and g are given continuous functions, and u is the solution to be determined. Most numerical methods in literature do not work well when the functions k and g have derivative singularity at the endpoints. To overcome the difficulty, Rashidinia–Zarebnia [2] proposed a Sinc-collocation method, which was derived without assuming differentiability at the endpoints. Okayama et al. [1] reformed their method and prove that it can attain $\mathrm{O}(\exp(-c\sqrt{N}))$, where N (approximately) denotes half the number of collocation points. The method is based on the Sinc approximation combined with the SE transformation: $t = \psi_1(x) = (b-a)\tanh(x/2)/2 + (b+a)/2$. They further improved the method by replacing SE transformation with the DE transformation: $t = \psi_2(x) = \psi_1(\pi \sinh x)$, and proved that the improved method can attain higher convergence rate: $\mathrm{O}(\exp(-cN/\log N))$.
However, those methods are a bit stressful to implement, especially in the case of system of Fredholm integral equations. This is because $(2N+1)$ collocation points t_i are taken as

$$t_i = \begin{cases} a & (i = -N) \\ \psi_\ell(ih) & (i = -N+1, \ldots, N-1) \\ b & (i = N) \end{cases}$$

and not consistent at $i = -N$ and $i = N$. To remedy this issue, this study derives another Sinc-collocation methods with the consistent collocation points

$$t_i = \psi_\ell(ih) \quad (i = -N, \ldots, N).$$

Furthermore, this study proves that the new method with $t = \psi_1(x)$ can attain $\mathrm{O}(\exp(-c\sqrt{N}))$, and the new method with $t = \psi_2(x)$ can attain $\mathrm{O}(\exp(-cN/\log N))$. Numerical comparisons are also presented in the talk.

References

[1] T. Okayama, T. Matsuo, M. Sugihara, *Improvement of a Sinc-collocation method for Fredholm integral equations of the second kind*, BIT Numer. Math., 51 (2011), pp. 339–366.

[2] J. Rashidinia, M. Zarebnia, *Numerical solution of linear integral equations by using sinc-collocation method*, Appl. Math. Comput., 168 (2005), pp. 806–822.