A numerical method for Volterra integral equations based on equispaced nodes

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The talk deals with the numerical treatment of Volterra integral equations of the type

$$f(s) + \mu \int_0^s k(t,s) f(t)(t-s)^\alpha t^\beta dt = g(s), \quad s \in (0,1),$$

where f is the unknown function, k and g are given functions, $\mu \in \mathbb{R}$, and $\alpha, \beta \geq 0$.

Here we propose a Nyström method based on a quadrature formula whose knots are equally spaced points of $[0,1]$. The use of equidistant points is crucial in many engineering and mathematical physics problems which are modeled by Volterra equations. Indeed, very often the functions k and g are available only at a discrete set of equispaced nodes, e.g. as results of experiments or measurement on the field. In these cases the classical methods based on piecewise polynomial approximation offer a lower degree of accuracy, while the efficient procedures based on the zeros of orthogonal polynomials cannot be used.

The quadrature formula herein is obtained by means of the sequence $\{B_{m,\ell}(f)\}_m$ of the so-called Generalized Bernstein polynomials, where $B_{m,\ell}(f)$ is the ℓ iterated boolean sum of the classical Bernstein polynomial $B_m(f)$, i.e.

$$B_{m,\ell}(f) = f - (f - B_m(f))^\ell, \qquad \ell \in \mathbb{N}, \quad B_{m,1}(f) = B_m(f).$$

We note that $B_{m,\ell}(f)$ requires the samples of f at equispaced nodes as well as the "original" Bernstein polynomial $B_m(f)$. However, differently from $B_m(f)$, its rate of convergence improves as the smoothness of the function increases. Indeed, by approximating $f \in C^{2\ell}([0,1])$ by $\{B_{m,\ell}(f)\}_{m,\ell}$, it is $\|f - B_{m,\ell}(f)\|_\infty \sim \mathcal{O}(m^{-\ell})$.

Finally, stability and convergence of the Nyström method are proved in Hölder–Zygmund type spaces and some numerical tests are given to confirm the theoretical estimates.