A stable BIE method for Laplace's equation with Neumann boundary conditions in domains with piecewise smooth boundaries

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It is well known that, in classical potential theory, the Laplace equation with Neumann boundary conditions can be reduced to integral equations of the second kind defined on the boundary of the domain. In particular, in this talk we consider the exterior Neumann problem in a open bounded simply connected planar domain $\Omega \subset \mathbb{R}^2$ with a piecewise smooth boundary Γ

$$\begin{aligned} \Delta u(x) &= 0, \qquad x \in \mathbb{R}^2 \setminus \bar{\Omega}, \\ \frac{\partial u(x)}{\partial \nu_x} &= f, \qquad x \in \Gamma, \\ |u(x)| &= o(1), \quad \text{as } |x| \to \infty, \end{aligned}$$

where ν_x denotes the outward-pointing unit normal vector to Γ at x.
Using the single layer representation of the potential

$$u(x) = -\int_\Gamma \psi(y) \log|x-y| dS(y), \quad x \in \mathbb{R}^2 \setminus \bar{\Omega},$$

the differential problem is reformulated in terms of the boundary integral equation (BIE)

$$-\pi\psi(x) - \int_\Gamma \frac{\partial}{\partial \nu_x} \log|x-y|\psi(y) dS(y) = f(x), \qquad x \in \Gamma,$$

whose solution ψ is the single layer density function and has singularities near the corners of the boundary.
A Nyström type method based on a proper Gauss-Jacobi-Lobatto quadrature formula is proposed for its approximation. Taking into account the known behavior of the solution, the analysis is carried out in proper weighted spaces of continuous functions.
Introducing a modification of the method in the vicinity of the corners we are able to prove both convergence and stability in such spaces.
The efficiency of the method is shown by illustrating some numerical tests.

References

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