An averaged Nyström method for 2D Fredholm integral equations

Patricia Díaz de Alba [a], Luisa Fermo[b], Giuseppe Rodriguez[b]

[a] Department of Mathematics, University of Salerno (Italy)
[b] Department of Mathematics and Computer Science, University of Cagliari (Italy)
pdiazdealba@unisa.it, fermo@unica.it, rodriguez@unica.it

This talk deals with the numerical solution of the following equation defined on the square $\mathcal{S} := [-1,1] \times [-1,1]$

$$(I-K)f = g,$$

where f is the bivariate function to be recovered, I is the identity operator, g is a given right-hand side sufficiently smooth on $(-1,1) \times (-1,1)$ possibly with algebraic singularities on the boundary, and Kf is the integral operator. It is defined as

$$(Kf)(\mathbf{y}) = \int_{\mathcal{S}} k(\mathbf{x},\mathbf{y}) f(\mathbf{x}) w(\mathbf{x}) d\mathbf{x}, \quad \mathbf{x} = (x_1, x_2), \mathbf{y} = (y_1, y_2) \in \mathcal{S},$$

where k is a known kernel function defined on $\mathcal{S} \times \mathcal{S}$ and

$$w(\mathbf{x}) = w_1(x_1) w_2(x_2),$$

with

$$w_i(x_i) = (1-x_i)^{\alpha_i}(1+x_i)^{\beta_i} \quad \alpha_i, \beta_i > -1, \quad i = 1, 2.$$

The function w is the product of two classical Jacobi weights and then contains any algebraic singularities on the boundaries of the kernel.
A Nystrom-type method based on Gauss and anti-Gauss cubature rules is developed and an averaged Nyström interpolant is proposed to improve the accuracy of the solution. Numerical tests show the performance of the approach.

References

[1] D.P. Laurie, *Anti-Gaussian quadrature formulas*, Mathematics of Computation, **65** (1996), 739–747.

[2] D. Occorsio and M.G. Russo, *Numerical methods for Fredholm integral equations on the square*, Applied Mathematics and Computation, 218 (2011), pp. 2318–2333.

[3] P. Díaz de Alba, L. Fermo, and G. Rodriguez, *Solution of second kind Fredholm integral equations by means of Gauss and anti-Gauss quadrature rules*, Numerische Mathematik, 146(4) (2020), pp. 699–728.