E-BEM for the resolution of 2D interior elastodynamic problems

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The study of elastic wave propagation phenomena can find relevant applications in many fields such as mechanical engineering, physics and geological soil analysis, giving pulse, for the accurate resolution of the inherent differential problem, to the study and the implementation of suitable numerical strategies. Among them, the Boundary Element Method (BEM) stands out as a powerful numerical tool, whose implementation requires to rewrite the physical displacement or traction as unknown of a Boundary Integral Equation (BIE) defined just at the boundary of the propagation domain, leading to a dimensional reduction of the initial problem. The BIE can be reformulated through energy arguments into a weak form, that we numerically solve after a space-time discretization of Galerkin type (E-BEM). This energetic method has been successfully implemented for the analysis of scalar wave propagation problems [2], while recent advances in 2D exterior elastodynamic soft scatterings can be found in [1, 3], where several numerical results are reported as validation of the E-BEM long time stability properties. In this contribution we will extend the outcomes obtained in the elastodynamic framework, with the purpose of treating Neumann and mixed boundary condition problems for the approximation of the resultant displacement in interior bounded domains. We will provide an overview of the possible combinations of the energetic weak formulations, that, depending on specific integral operators with kernels defined as spatial derivatives of the 2D elastodynamic Green tensor, allows us to incorporate the boundary conditions in the weak problem. Algorithmic considerations about the discretization of the obtained weak formulations will be included in the contribution, with a focus on the quadrature strategies required for the computation of matrix entries featured by different type of spatial singularities. In the end, we will present numerical tests concerning the implementation of the method, highlighting its efficiency and the long-time stability of the approximated solutions obtained solving by E-BEM for the considered elastodynamic problems.

References

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