Energetic Boundary Element Method for 3D wavefield modelling

A. Aimi[a], S. Dallospedale[a], **L. Desiderio**[a], C. Guardasoni[a]

[a] Department of Mathematical, Physical and Computer Sciences, University of Parma (Italy)
alessandra.aimi@unipr.it, simone.dallospedale@studenti.unipr.it,
luca.desiderio@unipr.it, chiara.guardasoni@unipr.it

We consider elastodynamic (vector) wave equation, defined in unbounded domains external to 3D bounded ones, endowed with null initial conditions and with a Dirichlet condition on the boundary. For its numerical solution, we reformulate the original differential problem in terms of a space-time Boundary Integral Equation (BIE) and then we employ a weak formulation linked to the energy of the system in order to achieve, in the approximation phase by means of a Galerkin-type Boundary Element Method (BEM), accurate and stable numerical results. This approach, called Energetic BEM (EBEM), leads to a linear system whose matrix has Toeplitz lower-triangular block structure, that allows the acceleration of the solution phase. As a direct consequence of the flexibility of the EBEM, a large body of literature has risen after the pioneering paper [1], to witness its capabilities to simulate 3D acoustic [2] and 2D elastodynamic [3] wave propagation in semi-infinite or infinite media. However, the extension of the EBEM to 3D elastic problems is not straightforward, since the energetic full space-time discretization requires double integration both in space and in time. Since a key ingredient for the success of the EBEM is the efficient and accurate evaluation of all the involved integrals, the selected formulation could be quite challenging in large scale applications. Nevertheless, if standard (constant) time and shape functions are employed, the double integration in time can be performed analytically and one is left with the task of evaluating double space integrals, whose integration domains are generally delimited by the wavefronts of the primary and the secondary waves. In order to exactly detect this latter, and consequently to preserve the stability properties of the EBEM, we choose boundary meshes made by triangular elements with straight sides and we propose an ad-hoc numerical integration scheme, tailored for the correct domain of integration.

References

[1] A. Aimi, M. Diligenti, C. Guardasoni, I. Mazzieri, S. Panizzi, *An energy approach to space–time Galerkin BEM for wave propagation problems*, Int. J. Numer. Meth. Engng., 80 (2009), pp. 1196–1240.

[2] A. Aimi, M. Diligenti, A. Frangi, C. Guardasoni, *Neumann exterior wave propagation problems: computational aspects of 3D energetic Galerkin BEM*, Comput. Mech., **51**(4), (2013), 475–493.

[3] A. Aimi, L. Desiderio, M. Diligenti, C. Guardasoni, *Application of Energetic BEM to 2D Elastodynamic Soft Scattering Problems*, Commun. Appl. Ind. Math., 10 (2019), pp. 182–198.