Filtered interpolation and the numerical resolution of systems of hypersingular integro-differential equations

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In this talk we show how a collocation-quadrature method based on de la Vallée Poussin filtered interpolation at Chebyshev nodes can be applied for solving systems of hypersingular integro-differential equations (HIDE) of the following type

$$\begin{aligned} \sigma\zeta_1(y) \quad + \quad a\zeta_2'(y) + \frac{b}{\pi}\int_{-1}^{1}\frac{\zeta_2(x)}{(x-y)^2}dx - \frac{1}{\pi}\int_{-1}^{1}\kappa_1(x,y)\zeta_2(x)dx = g_1(y), \\ y \in (-1,1), \\ \sigma\zeta_2(y) \quad + \quad a\zeta_1'(y) + \frac{b}{\pi}\int_{-1}^{1}\frac{\zeta_1(x)}{(x-y)^2}dx + \frac{1}{\pi}\int_{-1}^{1}\kappa_2(x,y)\zeta_1(x)dx = g_2(y), \end{aligned}$$

with $\sigma \in \mathbb{R}$, $\kappa_i(x,y)$ and $g_i(y), i = 1,2$, given functions, the constants $a, b \in \mathbb{R}$ s.t. $a^2+b^2 = 1$, and the unknown solution $\mathcal{Z} = (\zeta_1, \zeta_2)$ a differentiable function, satisfying the zero boundary condition

$$\mathcal{Z}(-1) = \mathcal{Z}(1) = 0.$$

The above systems are of interest because, for example, appear in the model describing the weak interface between two elastic materials containing a periodic array of micro-crazes [3]. Indeed, the boundary conditions yielding there to the solution of the posed problem are given in terms of an HIDE system.
The method is based on the procedure proposed in [2]. In the special case $\kappa_1 \equiv \kappa_2$ it is conveniently combined with a procedure presented in [1] that converts the system into a separable system of two independent equations. We prove its stability and uniform convergence in Hölder-Zygmund spaces of locally continuous functions and we show its efficiency through some numerical examples.

References

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