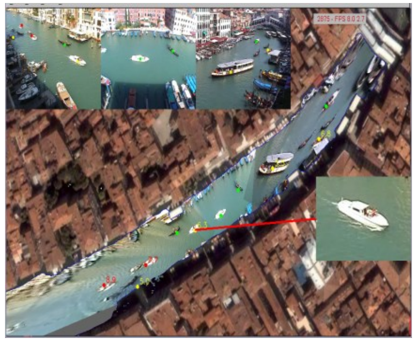
**UNIVERSITÀ DEGLI STUDI DELLA BASILICATA**

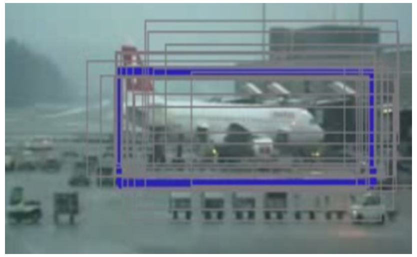
*Corso di Visione e Percezione*

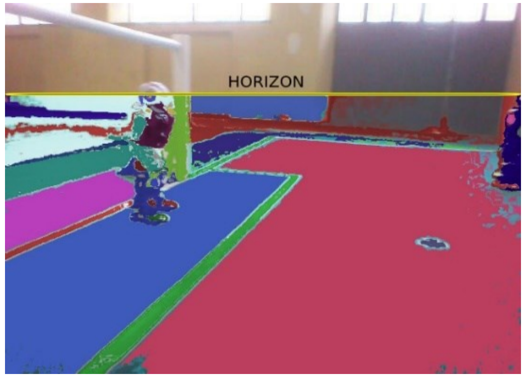
Docente

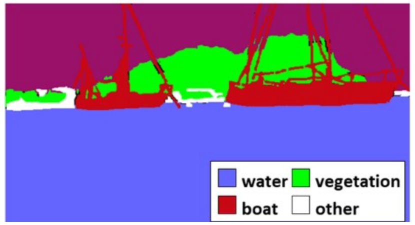
Domenico D. Bloisi



# Omografie



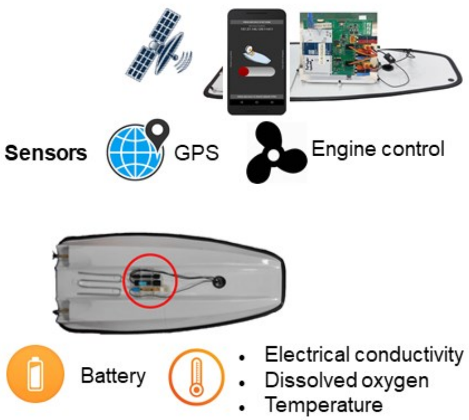


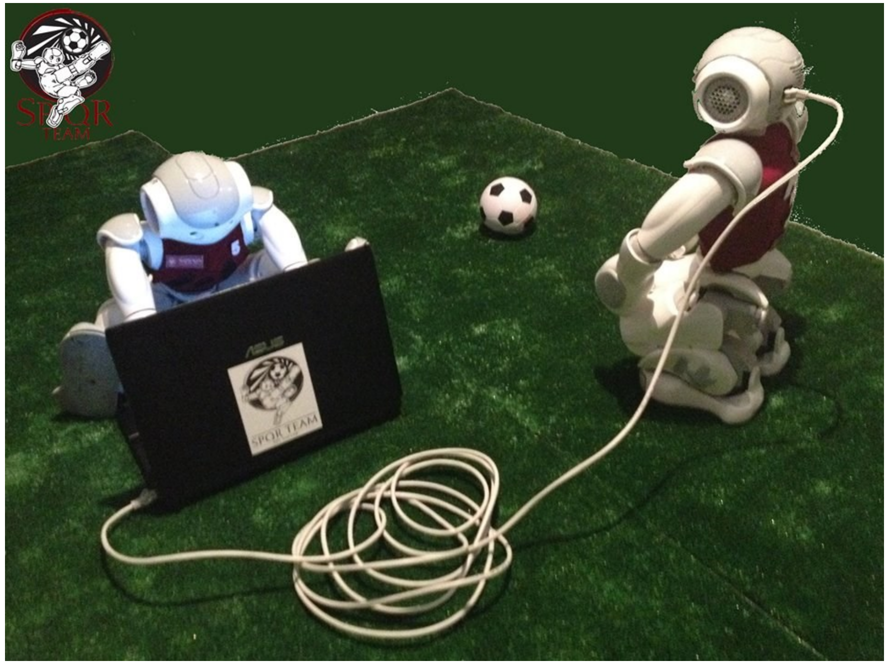


# Domenico Daniele Bloisi

- Ricercatore RTD B
  Dipartimento di Matematica, Informatica ed Economia
  Università degli studi della Basilicata
  [http://web.unibas.it/bloisi](http://web.unibas.it/bloisi)

- SPQR Robot Soccer Team
  Dipartimento di Informatica, Automatica e Gestionale Università degli studi di Roma "La Sapienza"
  [http://spqr.diag.uniroma1.it](http://spqr.diag.uniroma1.it)





# Informazioni sul corso

- Home page del corso
  http://web.unibas.it/bloisi/corsi/visione-e-percezione.html
- Docente: Domenico Daniele Bloisi
- Periodo: II semestre marzo 2021 – giugno 2021

  Martedì 17:00-19:00 (Aula COPERNICO)
  Mercoledì 8:30-10:30 (Aula COPERNICO)

Codice corso Google Classroom:
https://classroom.google.com/c/NjI2MjA4MzgzNDFa?cjc=xgolays

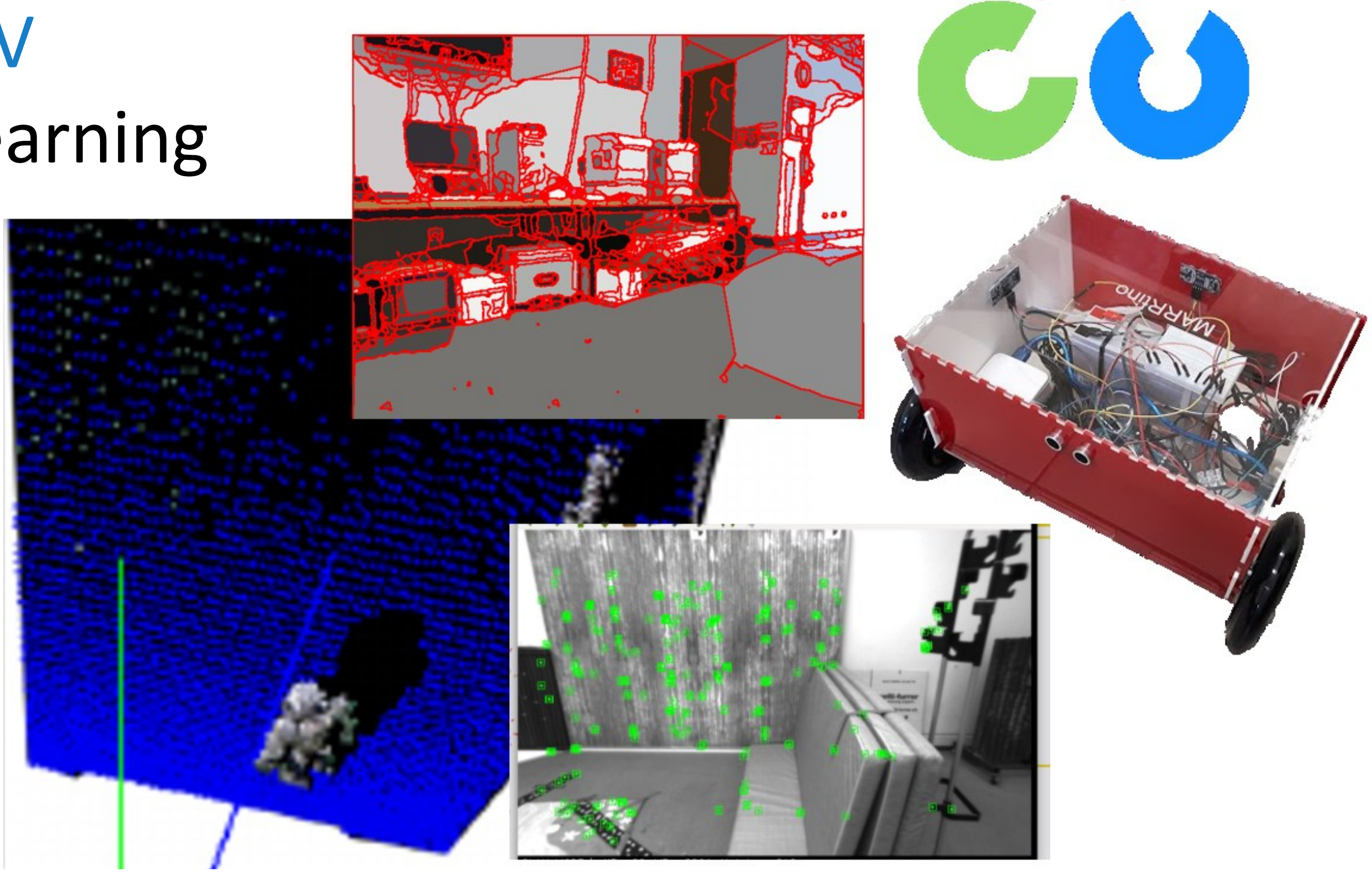
# Ricevimento

- Su appuntamento tramite Google Meet

Per prenotare un appuntamento inviare una email a domenico.bloisi@unibas.it

# Programma – Visione e Percezione

- Introduzione al linguaggio Python
- Elaborazione delle immagini con Python
- Percezione 2D – OpenCV
- Introduzione al Deep Learning
- ROS
- Il paradigma publisher and subscriber
- Simulatori
- Percezione 3D - PCL

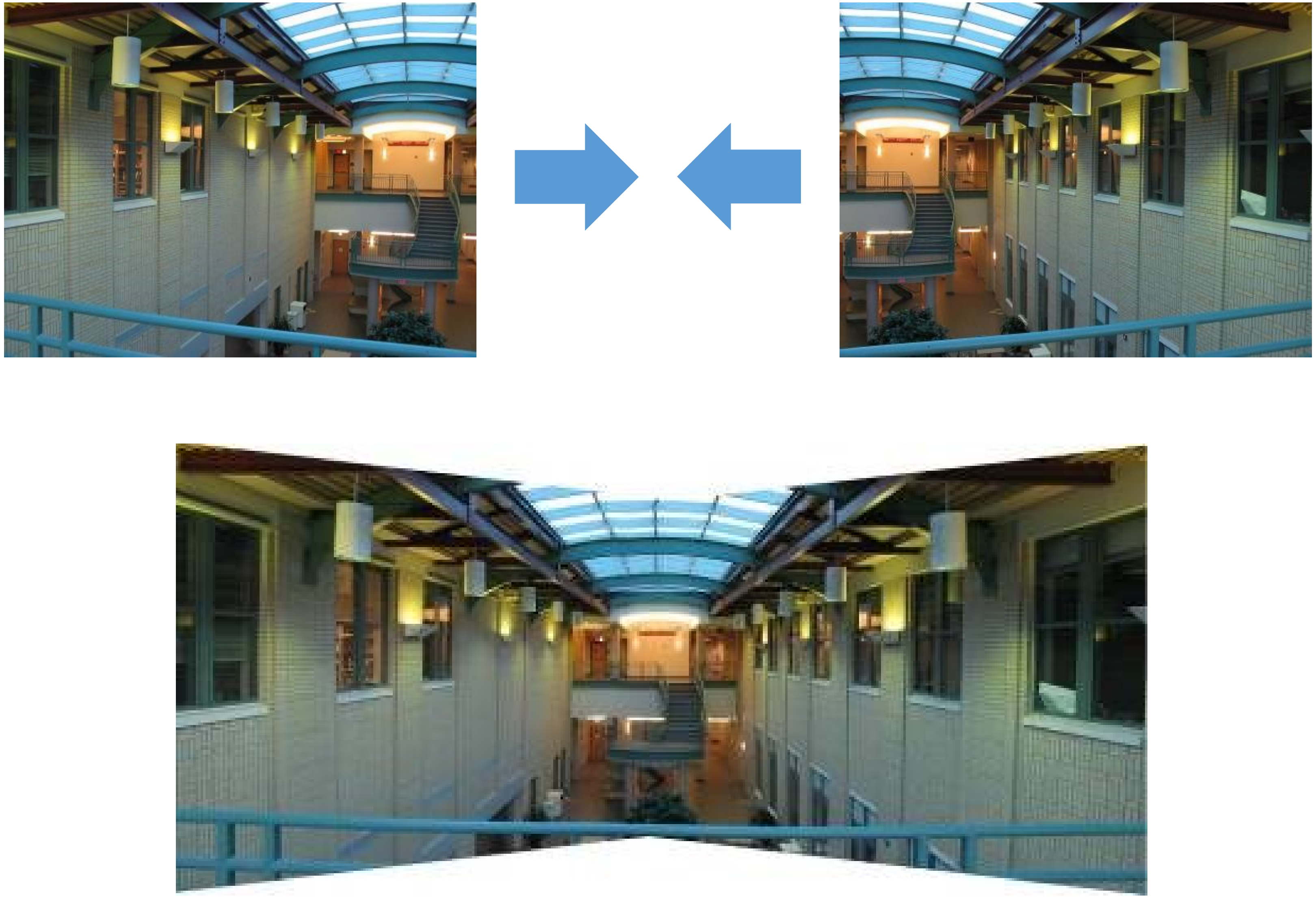


# Riferimenti

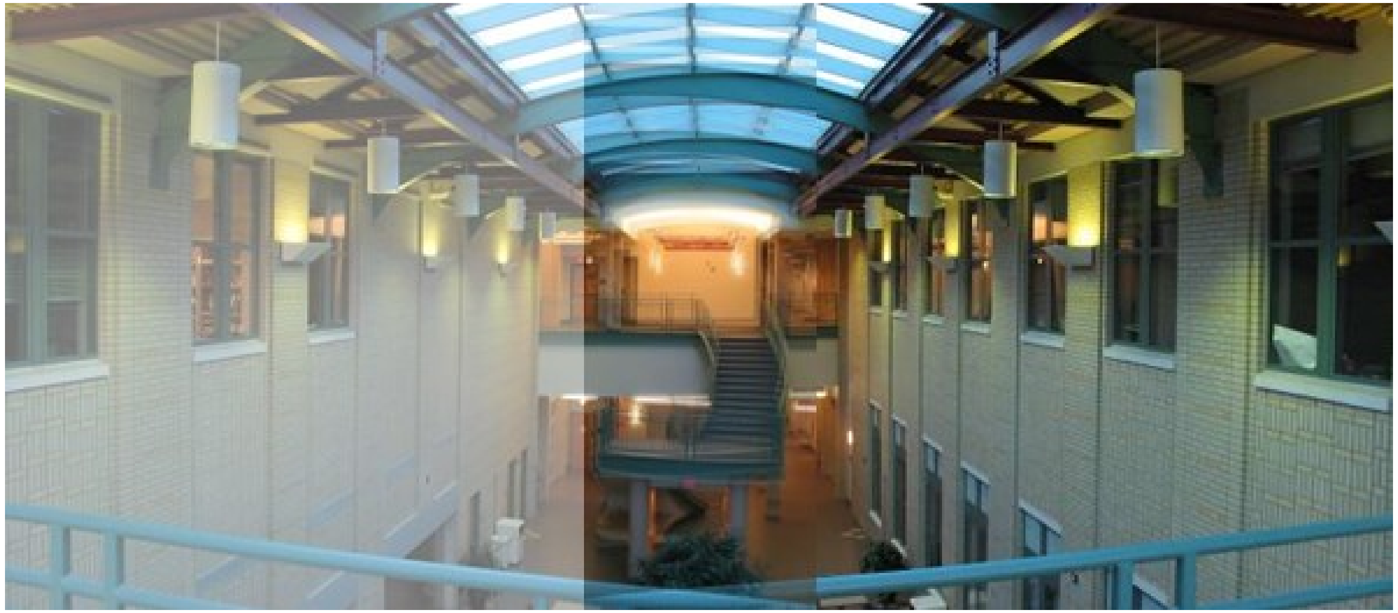
- Queste slide sono adattate da
  - Noah Snavely - CS5670: Computer Vision
    "Lecture 7: Transformations and warping“
  - M. Brown and D. G. Lowe
    Recognising Panoramas

- I contenuti fanno riferimento al capitolo 3 del libro "Computer Vision: Algorithms and Applications" di Richard Szeliski, disponibile al seguente indirizzo http://szeliski.org/Book/

# Image alignment



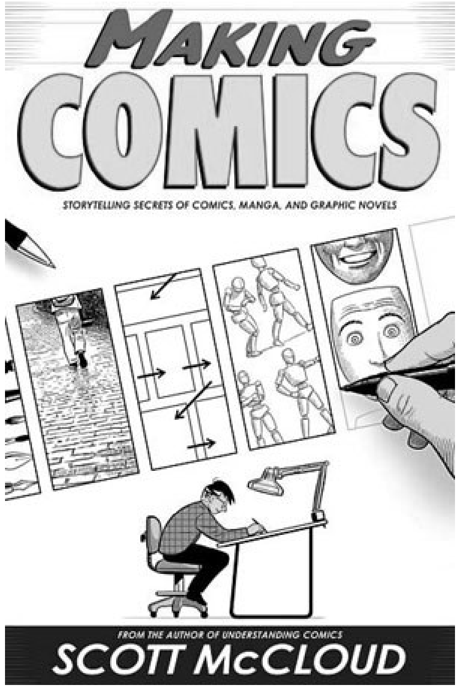
# Sovrapposizione



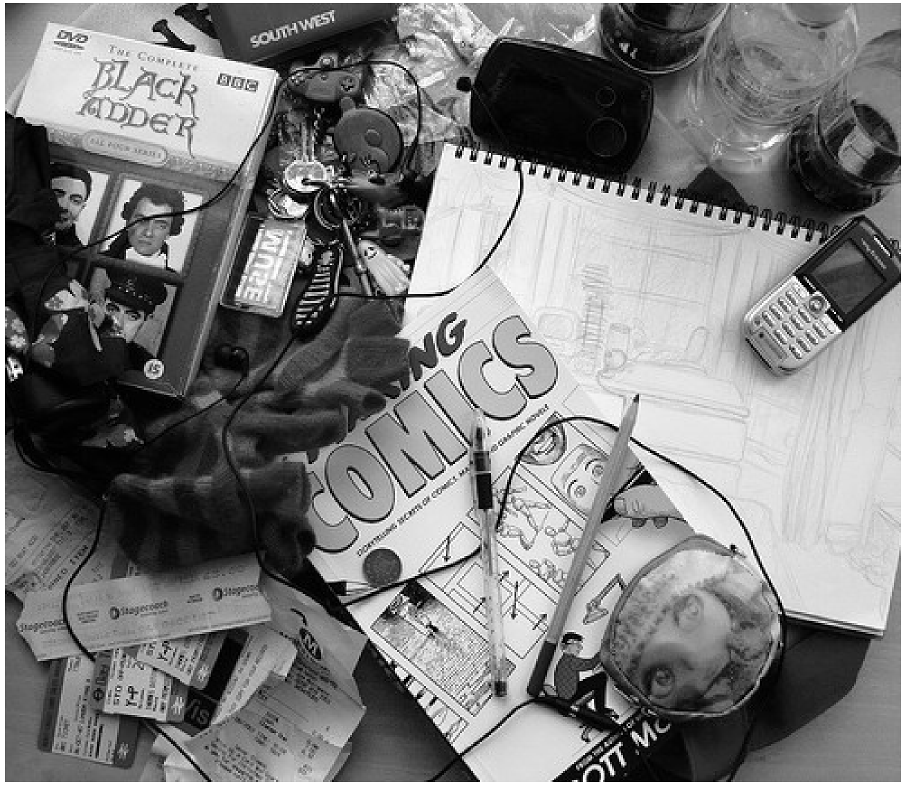
Non è un buon risultato!

# Image Transformation

What is the geometric relationship between these two images?

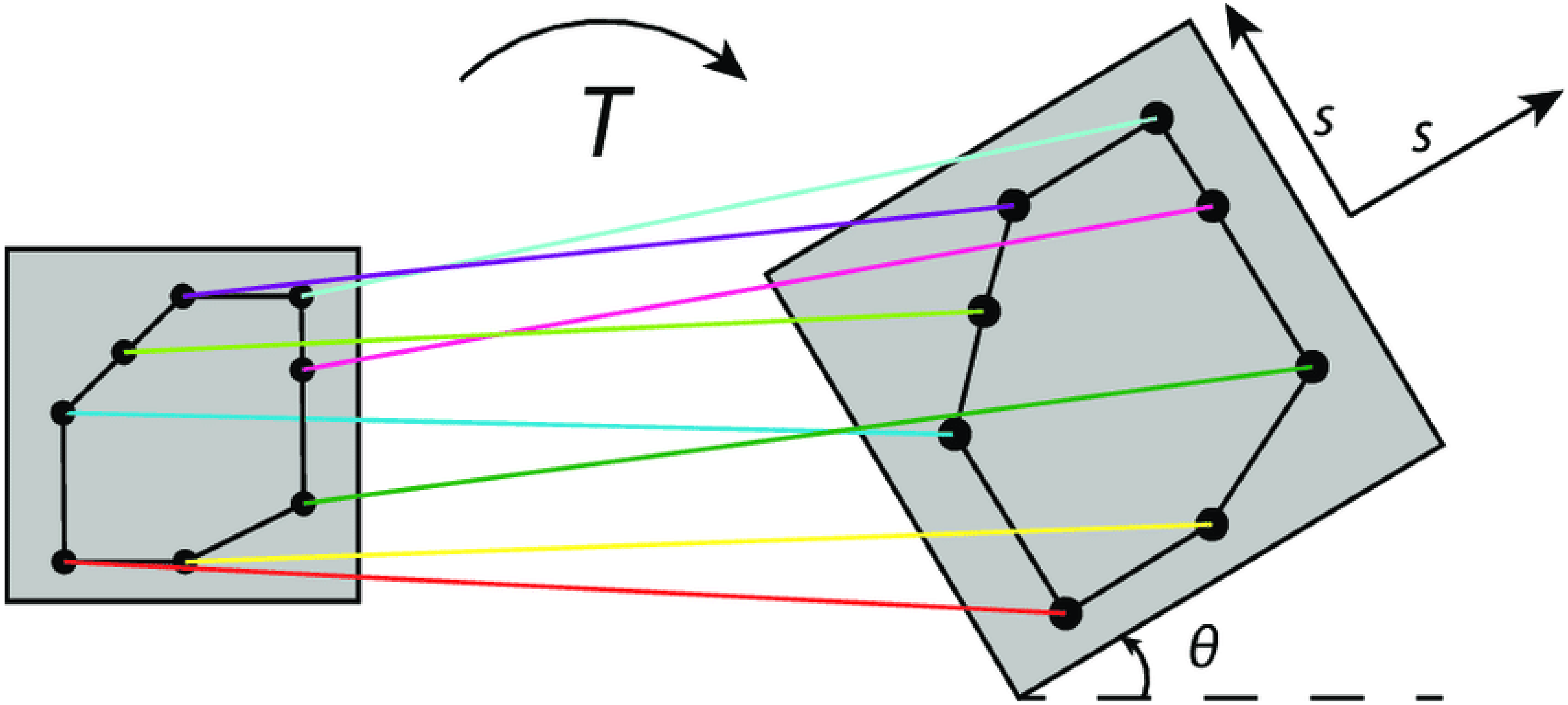

MAKING
COMICS
STORYTELLING SECRETS OF COMICS, MANGA, AND GRAPHIC NOVELS
FROM THE AUTHOR OF UNDERSTANDING COMICS
SCOTT McCLOUD

?



# Similarity Transformation

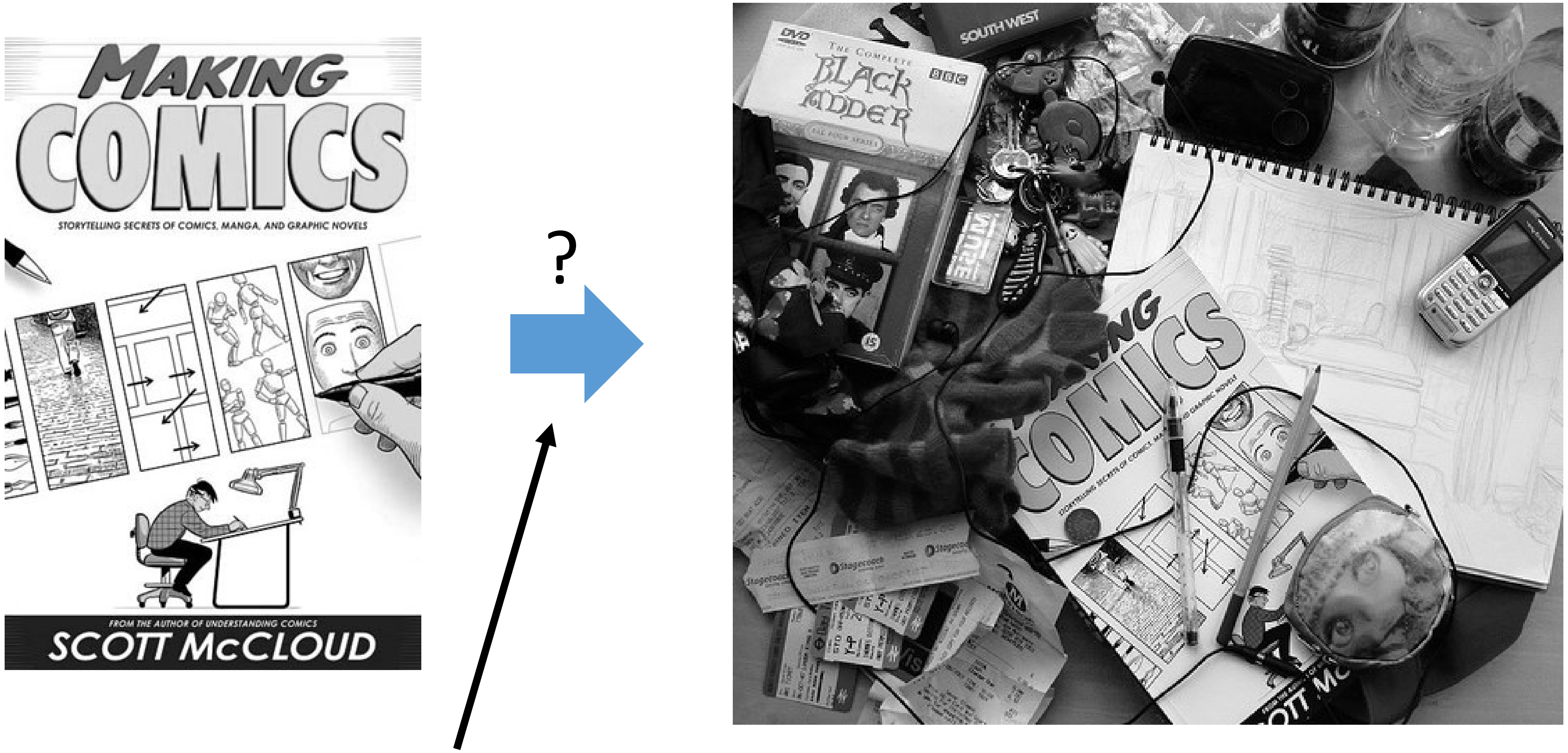
Translation + rotation + uniform scale



Source: Tobias Pahlberg

# Similarity Transformation

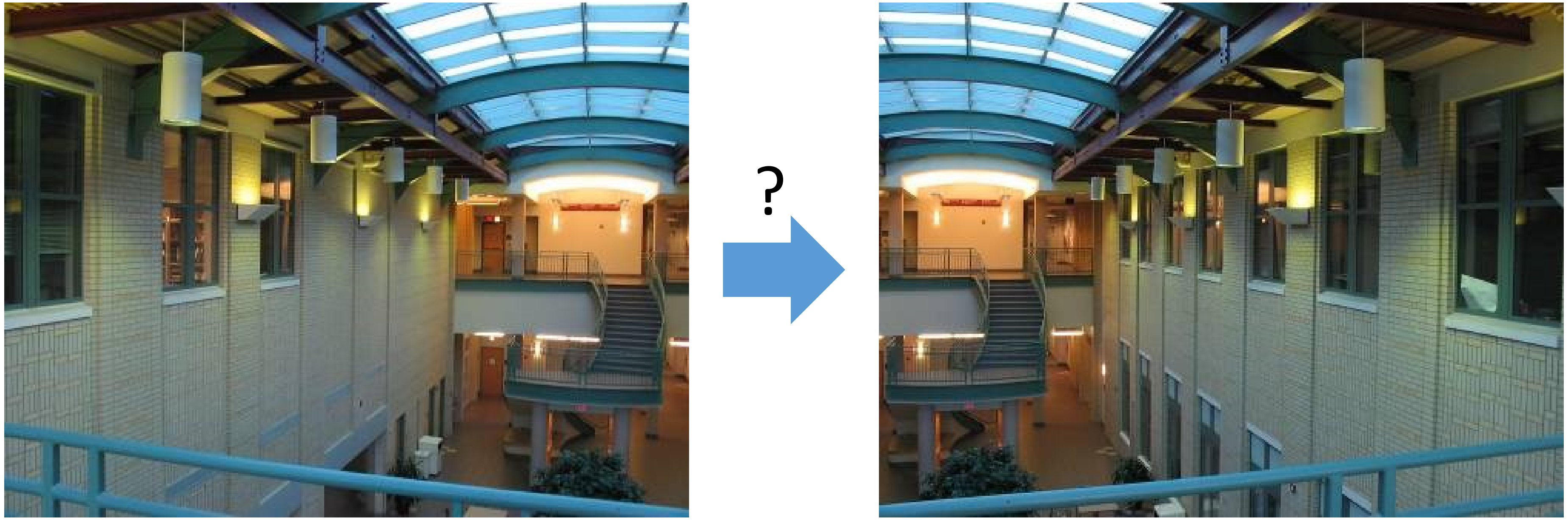
What is the geometric relationship between these two images?


MAKING
COMICS
STORYTELLING SECRETS OF COMICS, MANGA, AND GRAPHIC NOVELS
FROM THE AUTHOR OF UNDERSTANDING COMICS
SCOTT McCLOUD
?

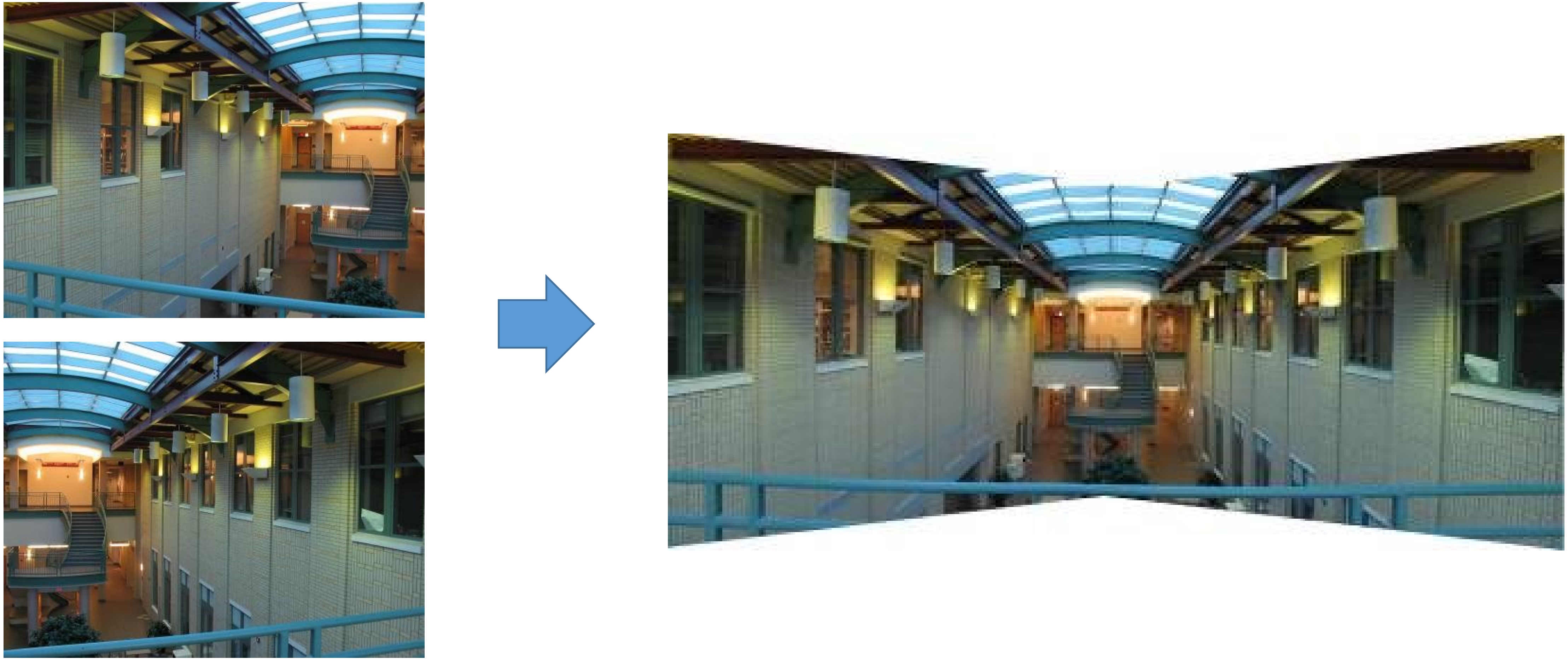
**Answer: Similarity transformation** (translation, rotation, uniform scale)

# Similarity?

What is the geometric relationship between these two images?

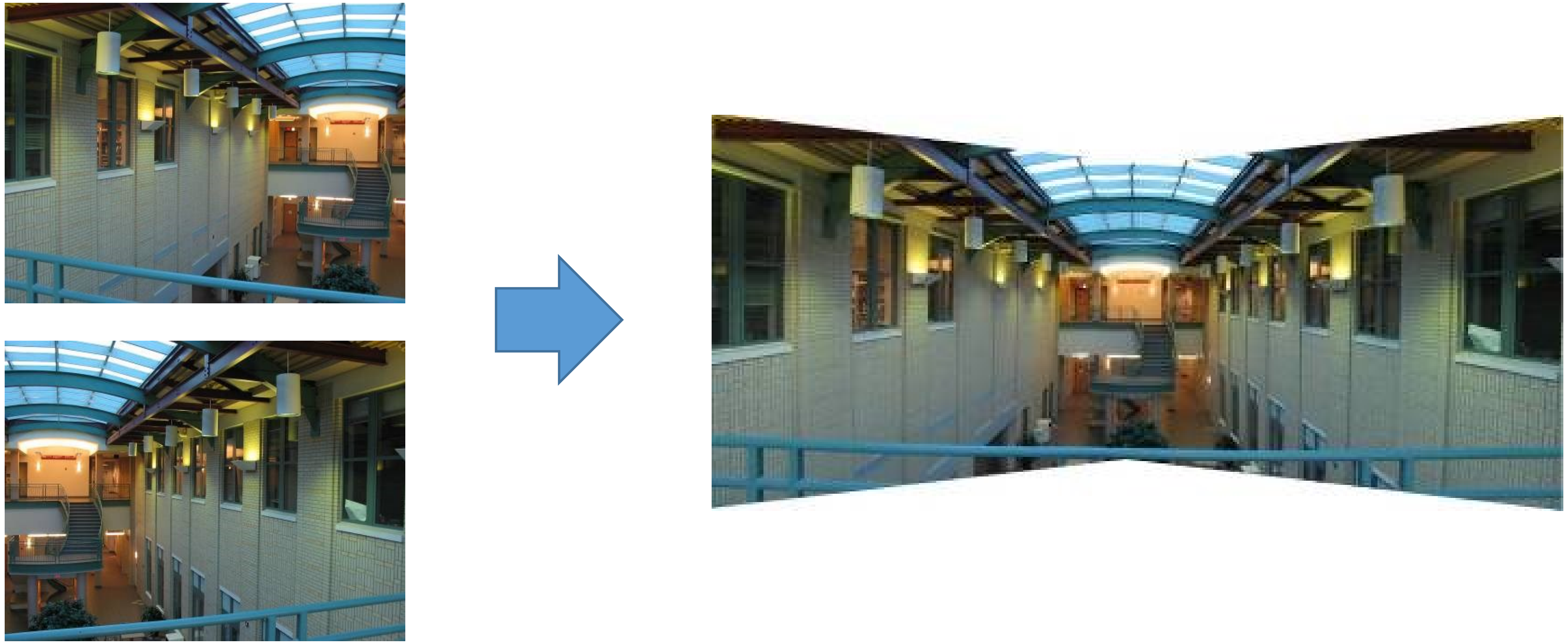


# Similarity?



**Non è una trasformazione simile!**

# Image Mosaicing

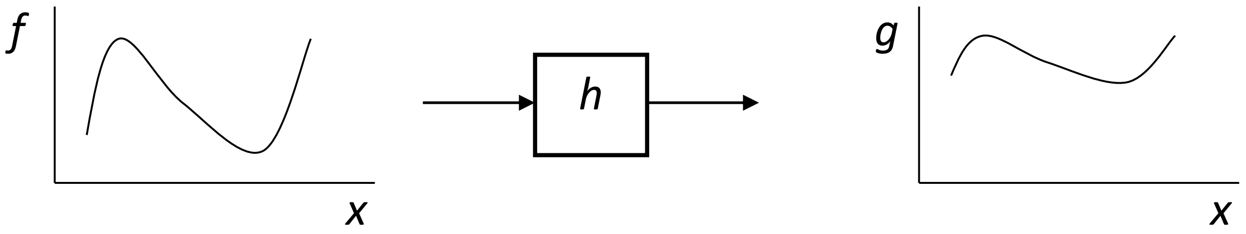


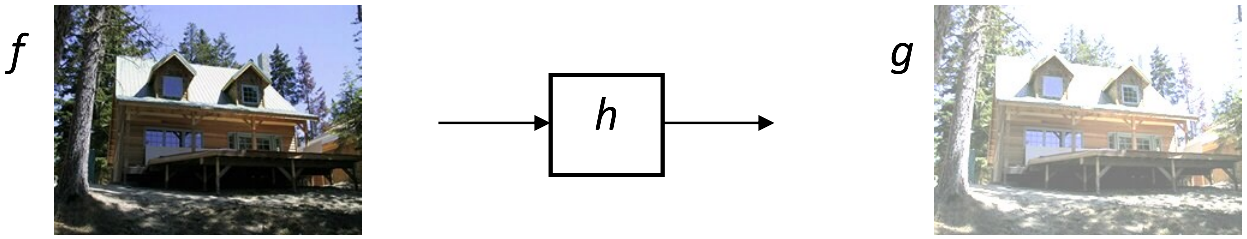
1. First, we need to know what this transformation is.
2. Second, we need to figure out how to compute it using feature matches.

# Image Filtering

- image filtering: change *range* of image

$$g(x) = h(f(x))$$

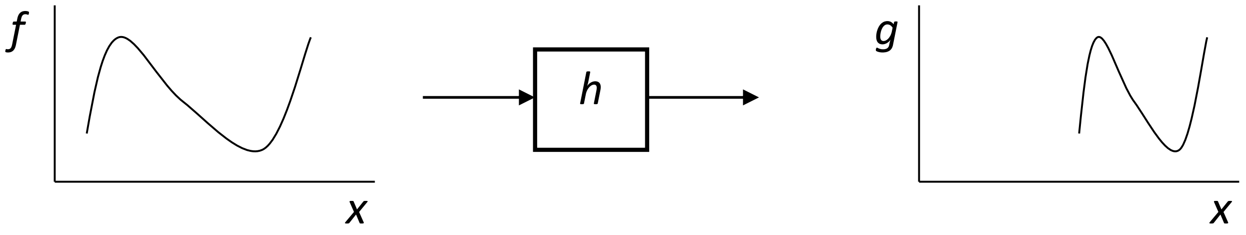


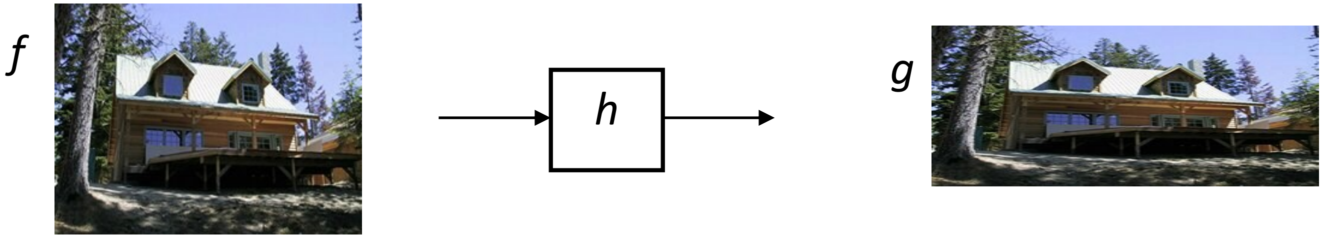


# Image Warping

- image warping: change *domain* of image

$$g(x) = f(h(x))$$

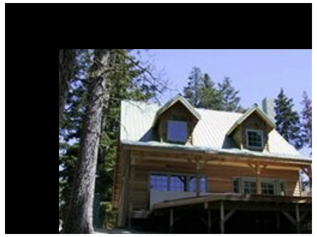




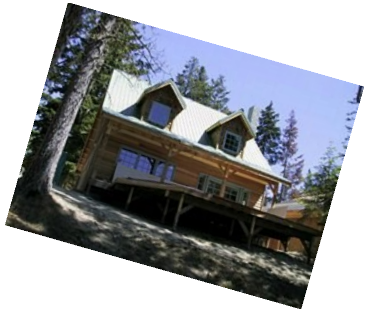
# Parametric (global) warping

Examples of parametric warps:

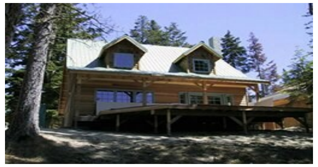
original



translation

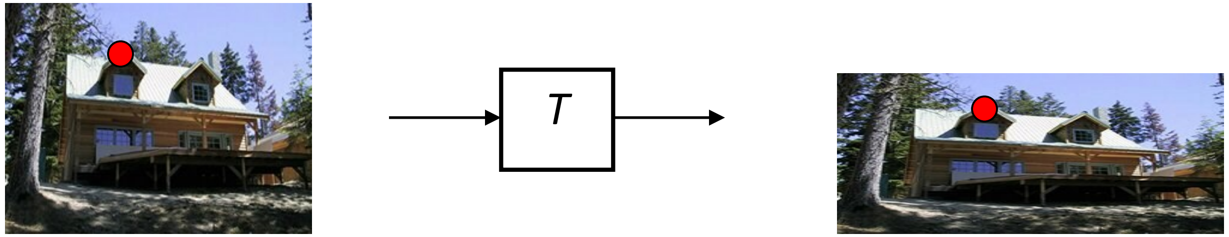


rotation



aspect

# Parametric (global) warping



**p** = (x,y)

**p’** = (x’,y’)

Transformation *T* is a coordinate-changing machine:

**p’** = *T*(**p**)

What does it mean that *T* is global?

- is the same for any point **p**
- can be described by just a few numbers (parameters)

# Linear transforms

Let's consider *linear* transforms

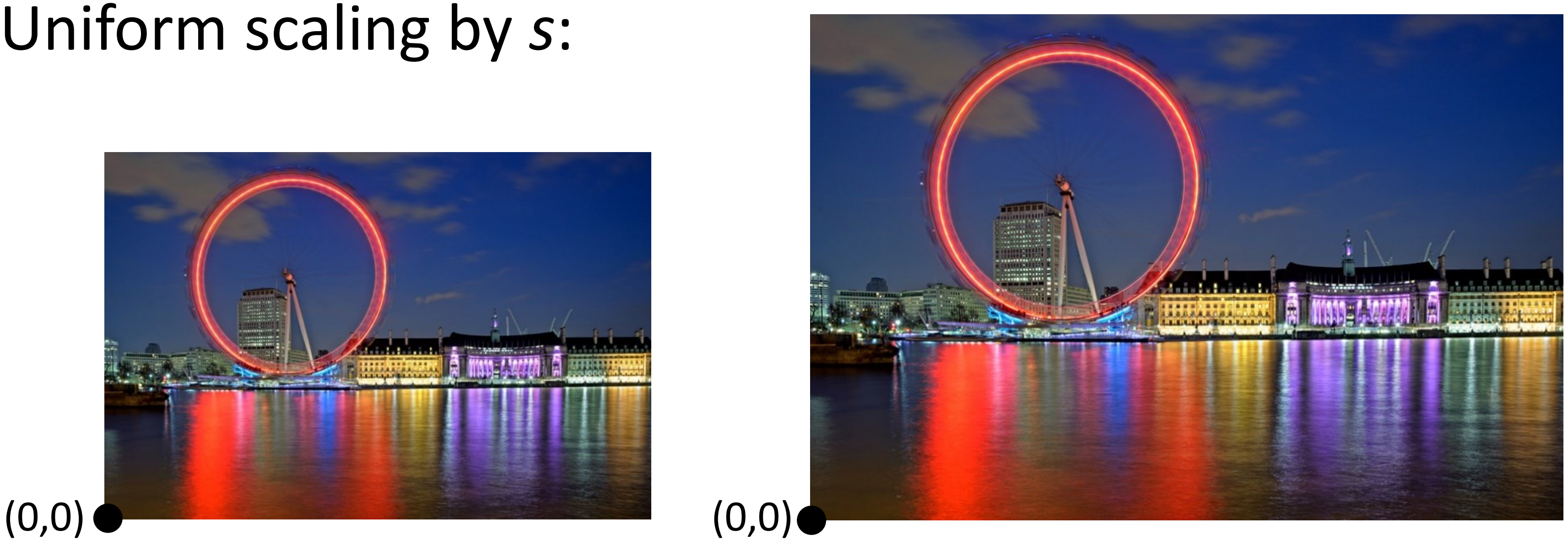
(can be represented by a 2x2 matrix):

$$\mathbf{p}' = \mathbf{T}\mathbf{p}$$

$$\left[\begin{array}{c} x' \\ y' \end{array}\right] = \mathbf{T}\left[\begin{array}{c} x \\ y \end{array}\right]$$

# Scaling

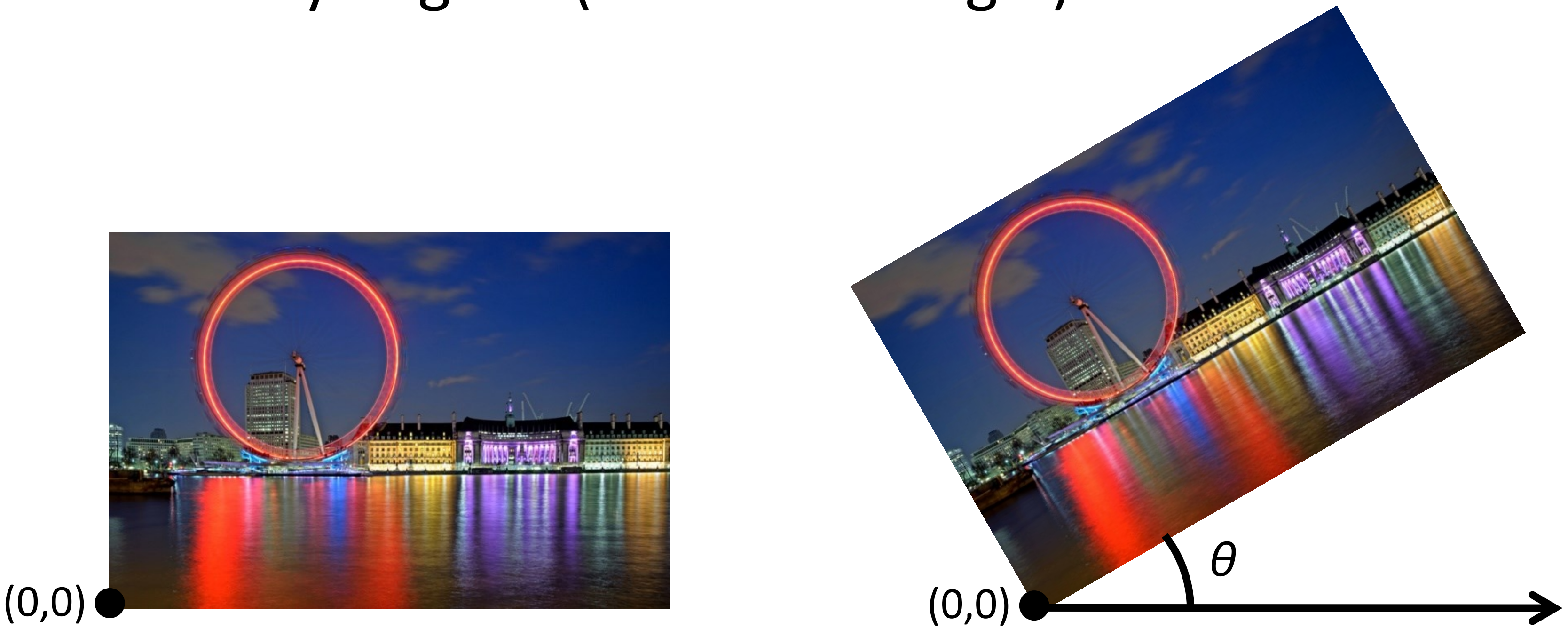
Uniform scaling by *s*:



$$\mathbf{S} = \left[ \begin{array}{cc} s & 0 \\ 0 & s \end{array} \right]$$

# Rotation

Rotation by angle θ (about the origin)



$$\mathbf{R} = \left[ \begin{array}{cc} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{array} \right]$$

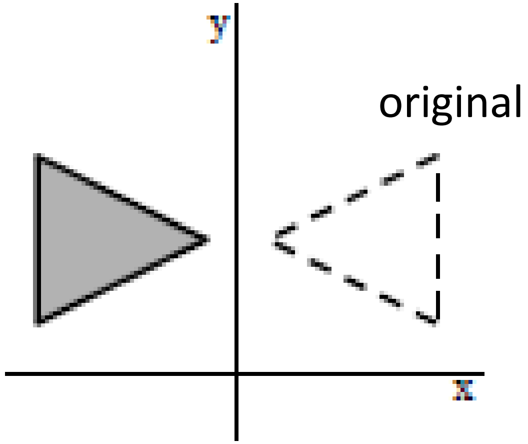
# 2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

## 2D mirror about Y axis?

$$\begin{aligned} x' &= -x \\ y' &= y \end{aligned}$$

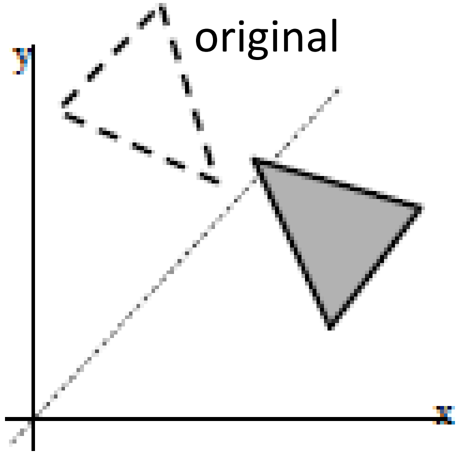
$$\mathbf{T} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



## 2D mirror across line y = x?

$$\begin{aligned} x' &= y \\ y' &= x \end{aligned}$$

$$\mathbf{T} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



# 2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Translation?

$$\begin{aligned} x' &= x + t_x \\ y' &= y + t_y \end{aligned}$$

NO!

Translation is not a linear operation on 2D coordinates

# All 2D Linear Transformations

Linear transformations are combinations of …

- Scale,
- Rotation,
- Shear, and
- Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

# Homogeneous Coordinates

How can we represent translation

$$\begin{aligned} x' &= x + t_x \\ y' &= y + t_y \end{aligned}$$

as a 3x3 matrix?

$$\begin{bmatrix} x \\ y \end{bmatrix} \xrightarrow{\text{homogeneous coords}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

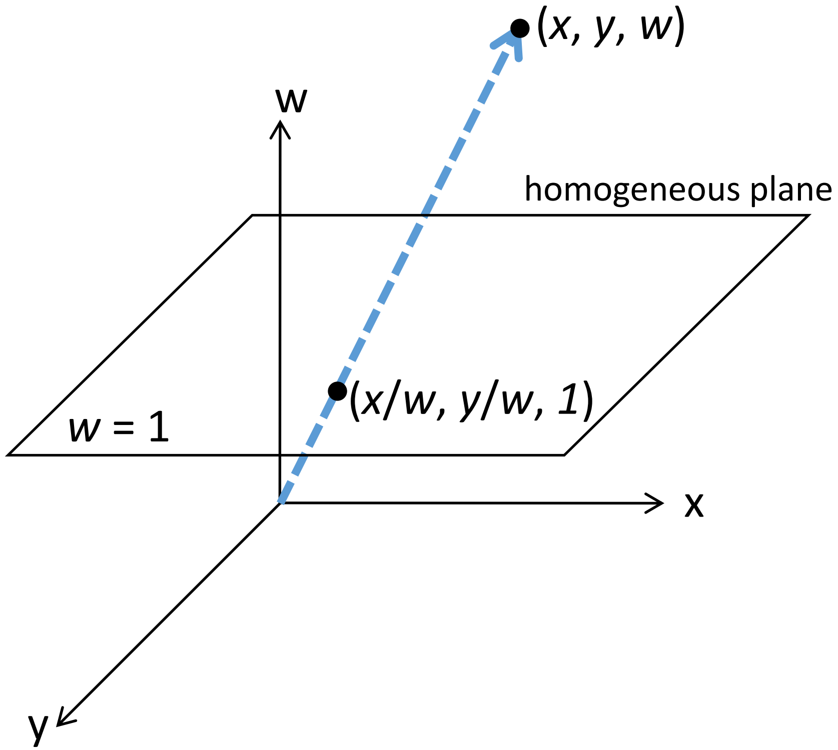
represent coordinates in 2 dimensions with a 3-vector

# Homogeneous coordinates

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates



Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

# Translation

Solution using homogeneous coordinates

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

# Affine transformations

$$\mathbf{T} = \left[\begin{array}{ccc} 1 & 0 & t_x \\ 0 & 1 & t_y \\ \hline \multicolumn{3}{|c|}{0 \qquad 0 \qquad 1} \\ \hline \end{array}\right]$$

any transformation represented by a 3x3 matrix with last row [ 0 0 1 ] we call an *affine transformation*

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$

# Affine transformations

Affine transformations are combinations of …

- Linear transformations, and
- Translations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

# Basic affine transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s_x} & 0 & 0 \\ 0 & \boldsymbol{s_y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Scale

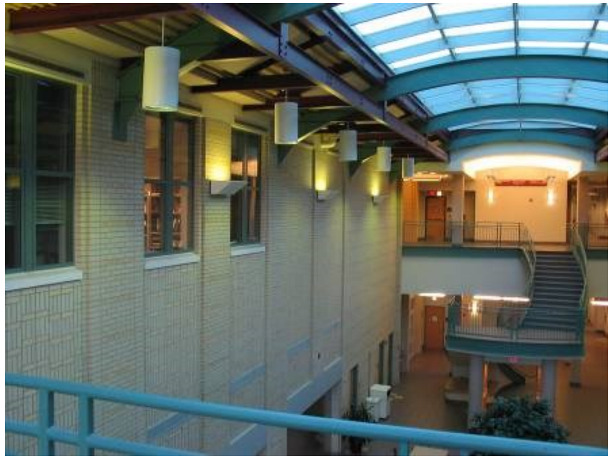
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

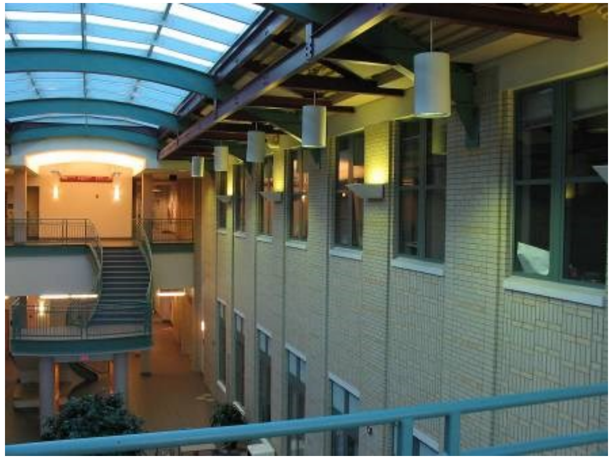
2D *in-plane* rotation

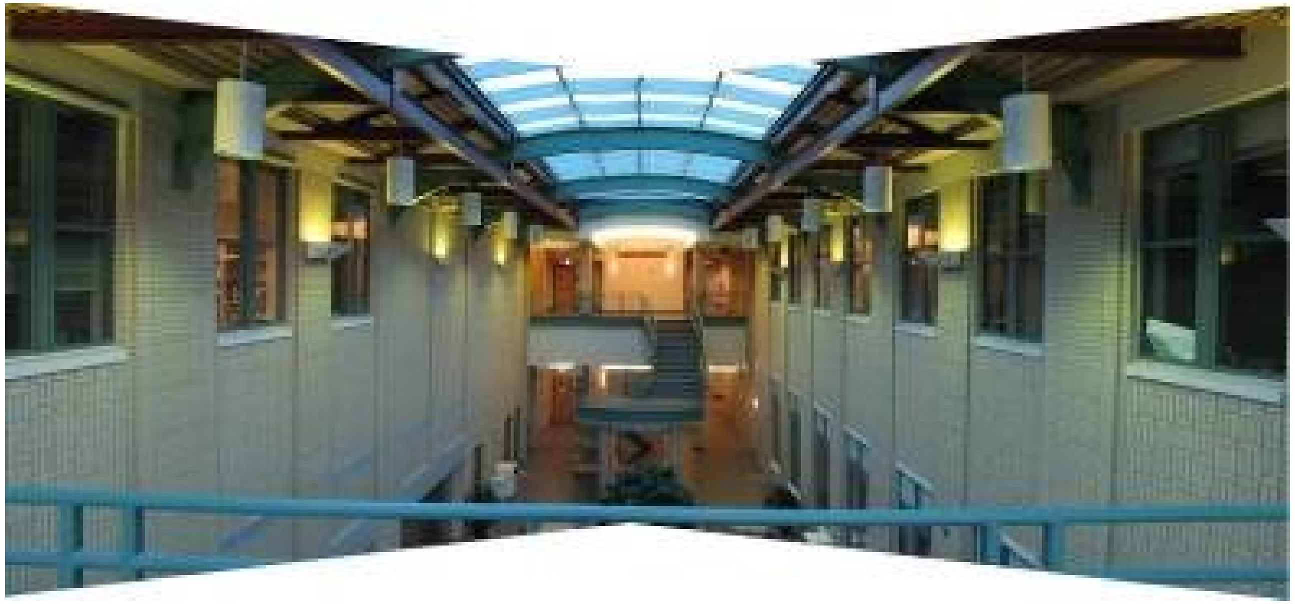
$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{sh_x} & 0 \\ \boldsymbol{sh_y} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Shear

# É una trasformazione affine?



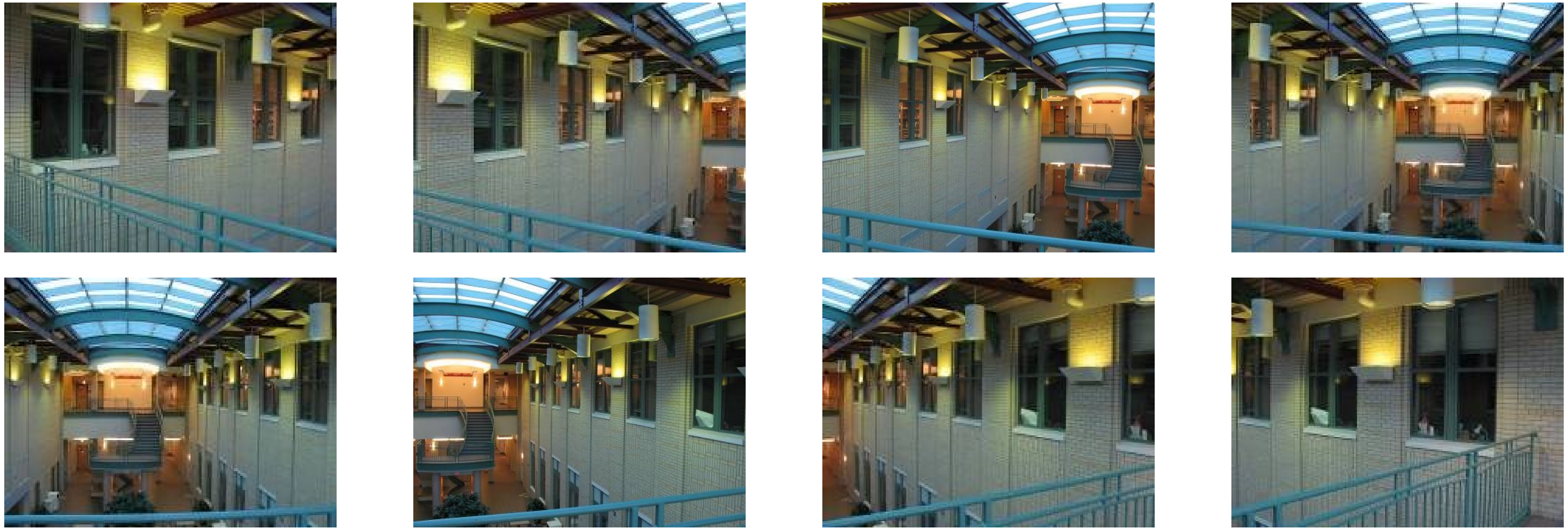


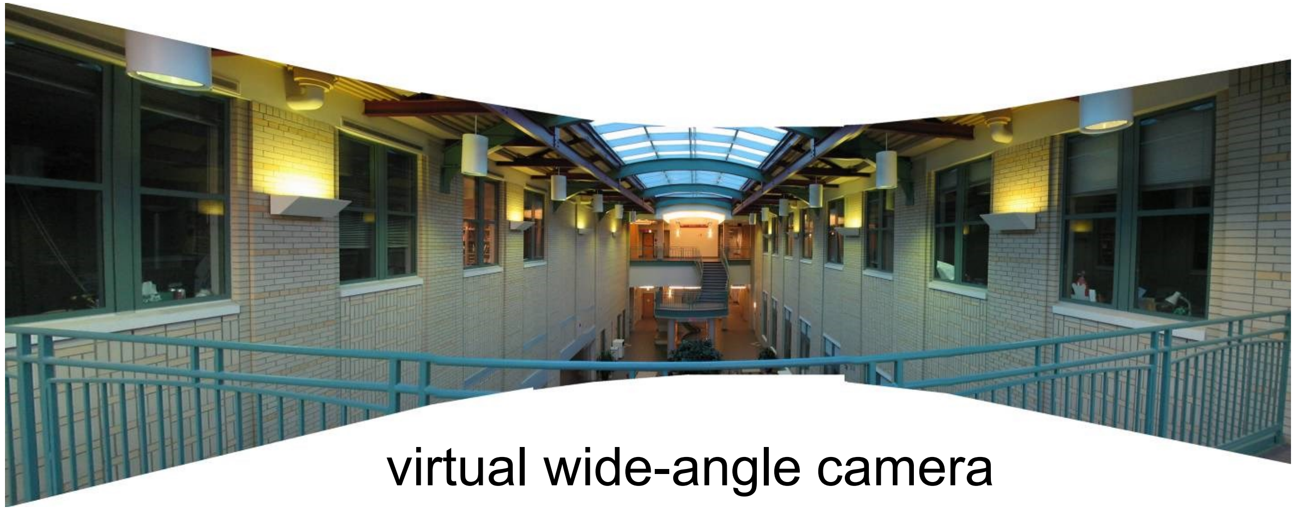


NO

- Parallel lines do not necessarily remain parallel
- Ratios are not preserved

# É una trasformazione affine?





virtual wide-angle camera

# Trasformazioni non affini

$$\begin{bmatrix} \begin{matrix} a & b & c \\ d & e & f \end{matrix} \\ \boxed{\begin{matrix} 0 & 0 & 1 \end{matrix}} \end{bmatrix}$$

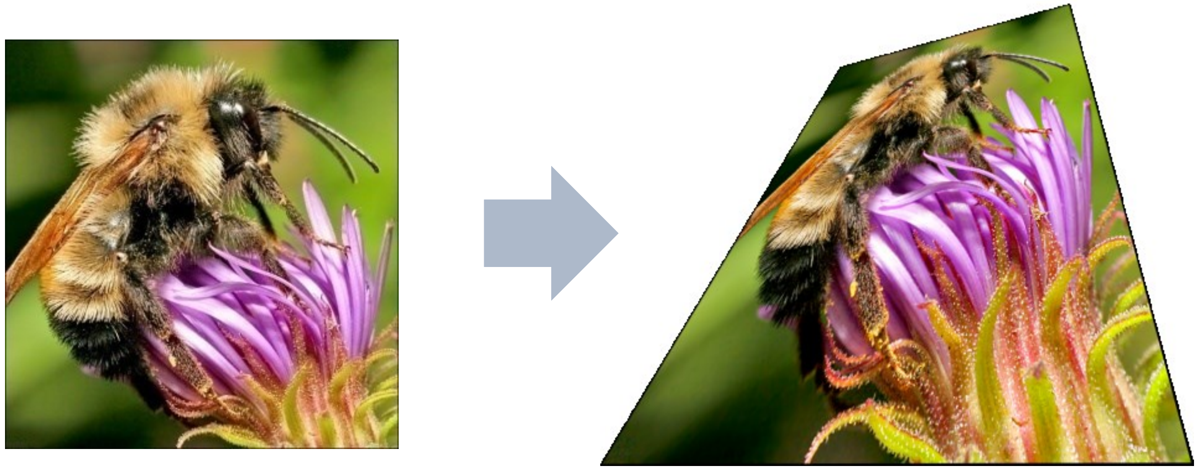
affine transformation

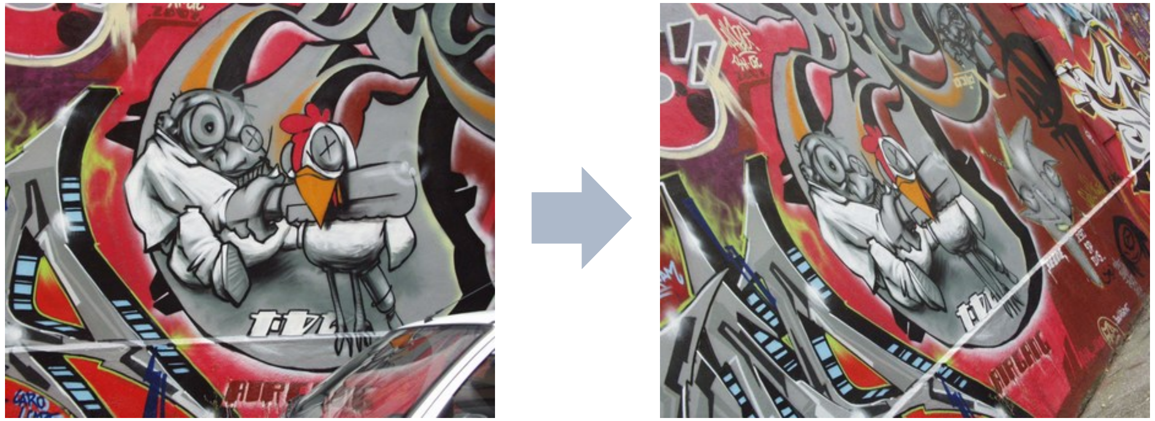
Cosa accade se cambiamo gli elementi della terza riga?

# Omografie

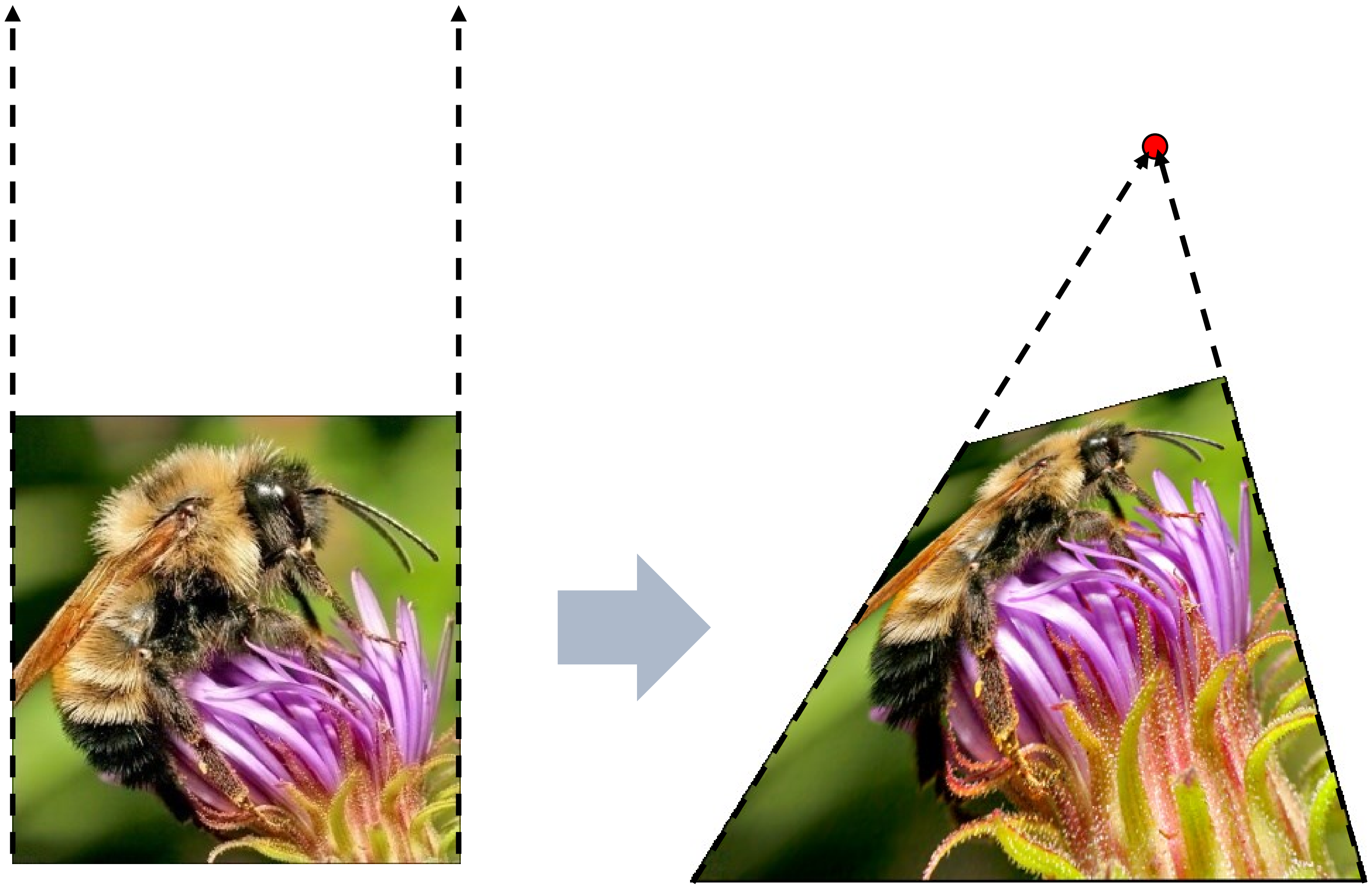
$$\mathbf{H} = \begin{bmatrix} a & b & c \\ d & e & f \\ \multicolumn{3}{c}{\boxed{g \quad h \quad 1}} \end{bmatrix}$$

H is a *homography* (or *planar perspective map*)





# Punti all’infinito



# Top view con omografie

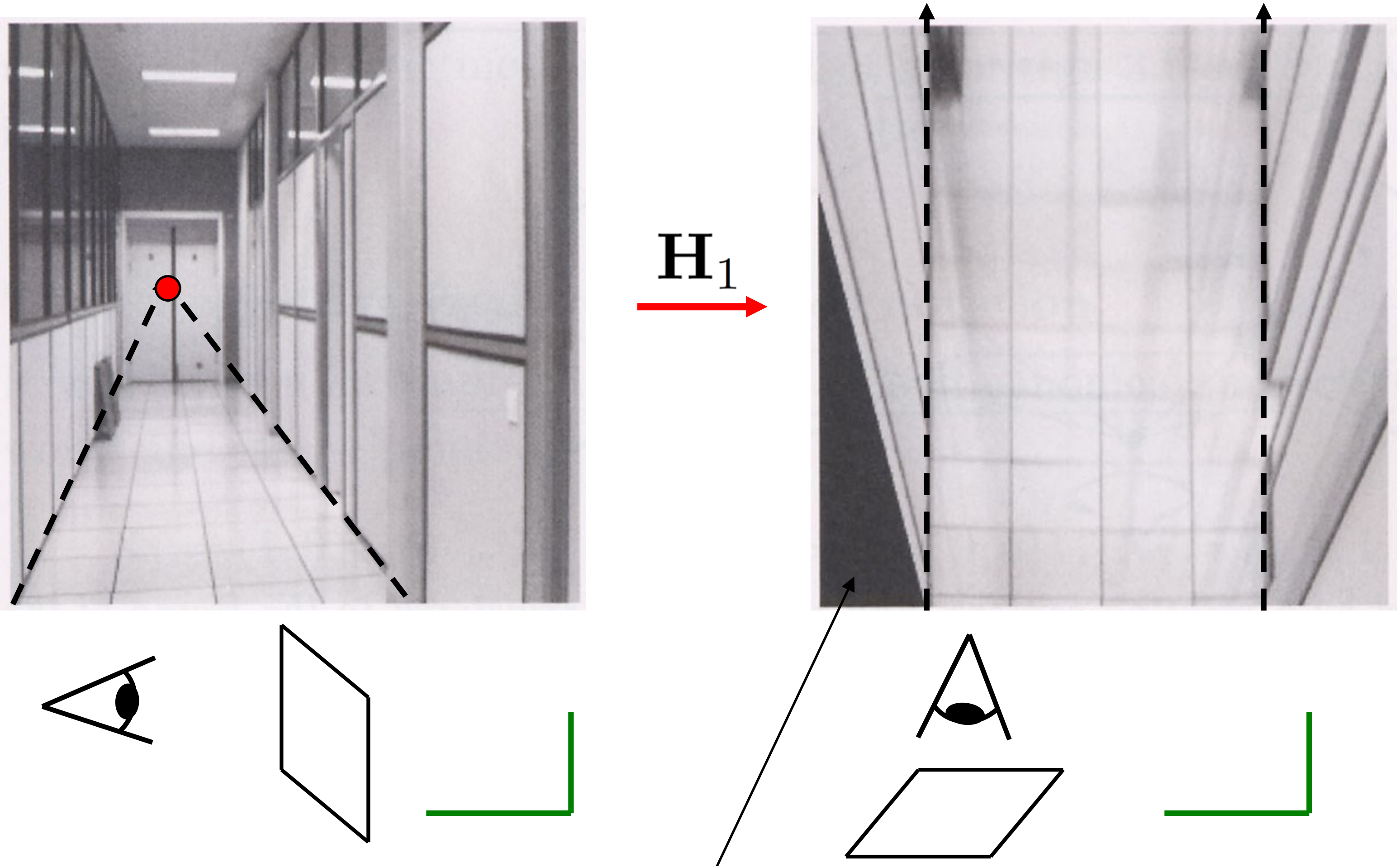
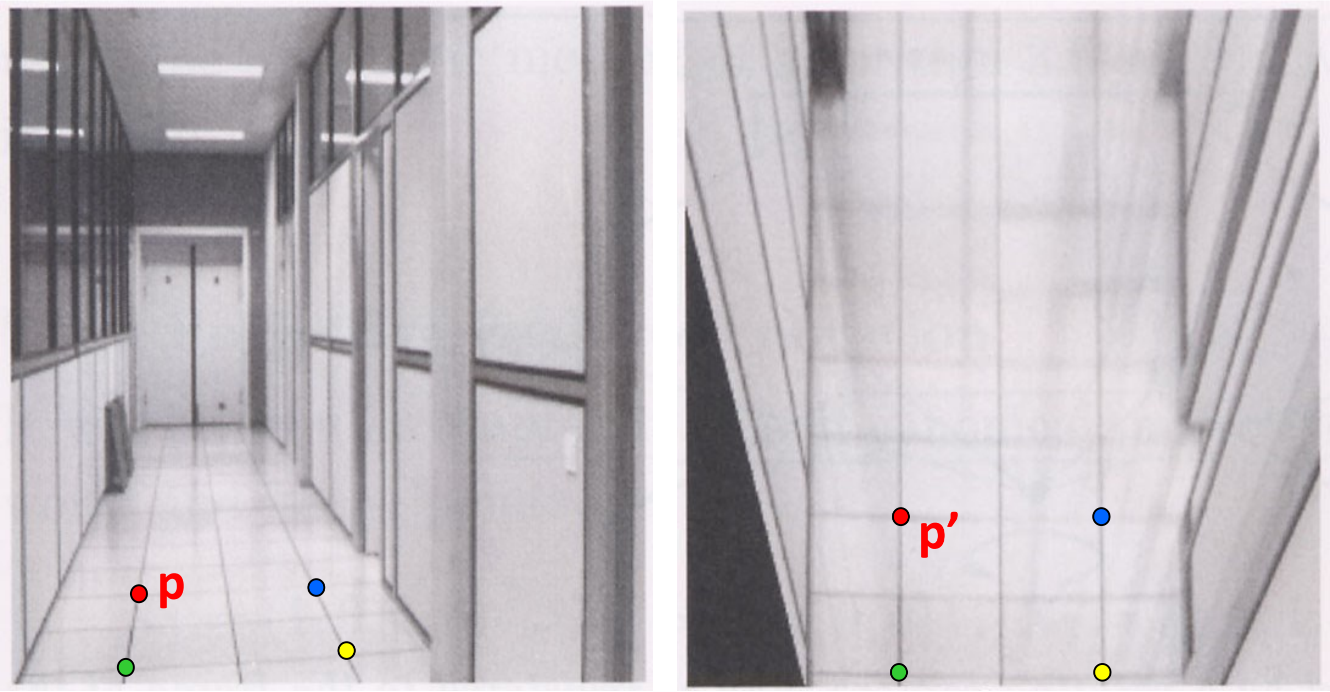


image plane in front

image plane below (top view)

black area where no pixel maps to

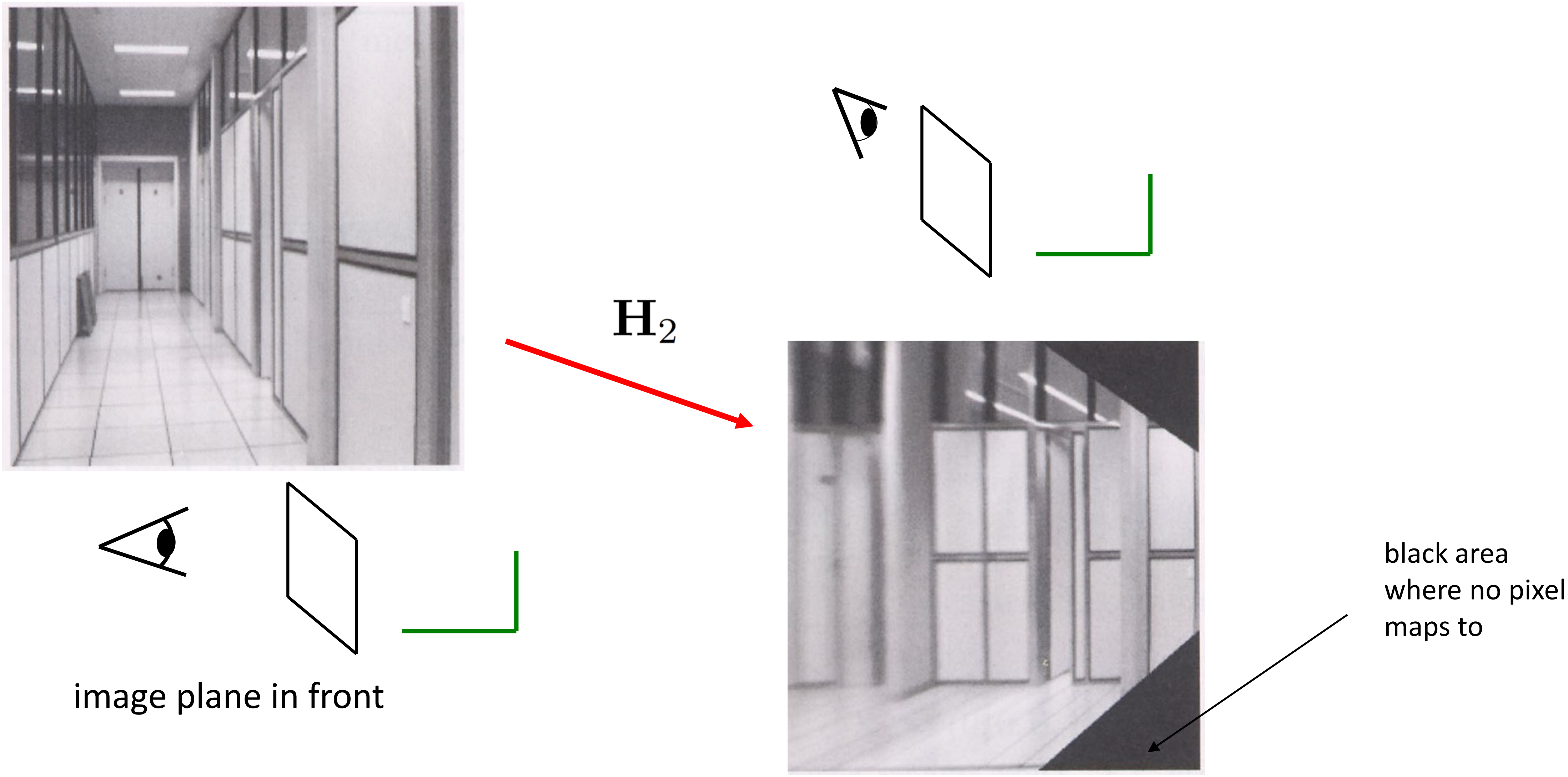
# Image rectification



To unwarp (rectify) an image

- Find the homography H given a set of p and p’ pairs
- How many correspondences are needed?
- Tricky to write H analytically, but we can solve for it!
- Find such H that “best” transforms points p into p’
- Use least-squares!

# Side view



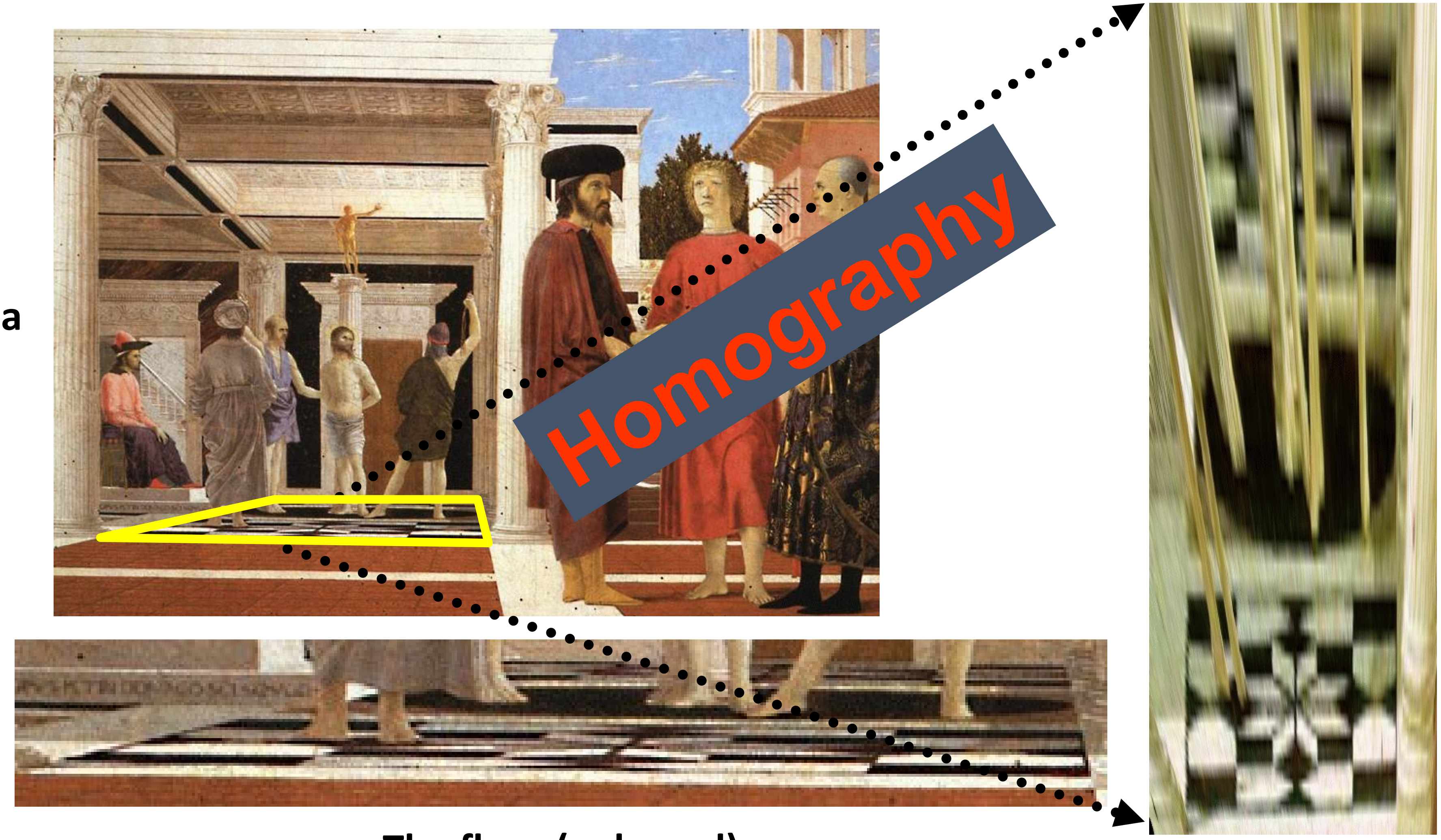
$\mathbf{H}_2$

image plane in front

black area
where no pixel
maps to

# What is the shape of the b/w floor pattern?

**Flagellation,**
**Piero della Francesca**



**The floor (enlarged)**

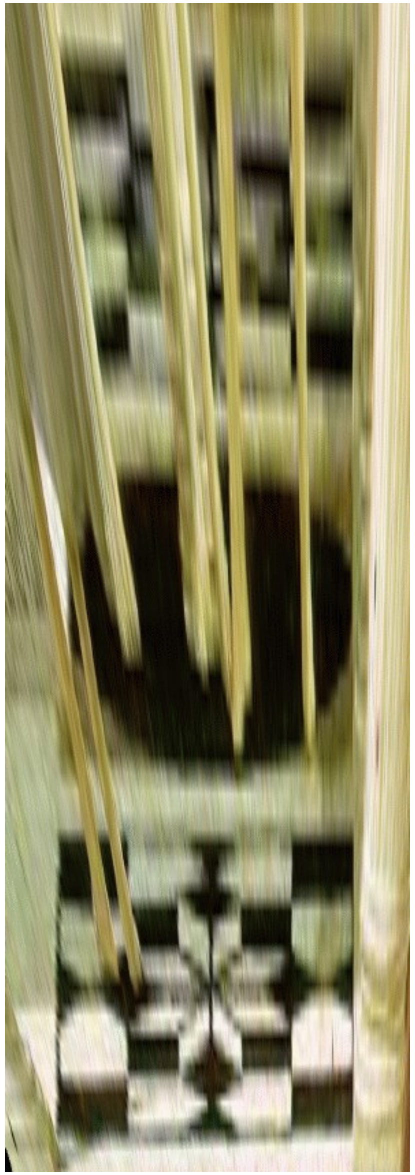
**automatically rectified floor**

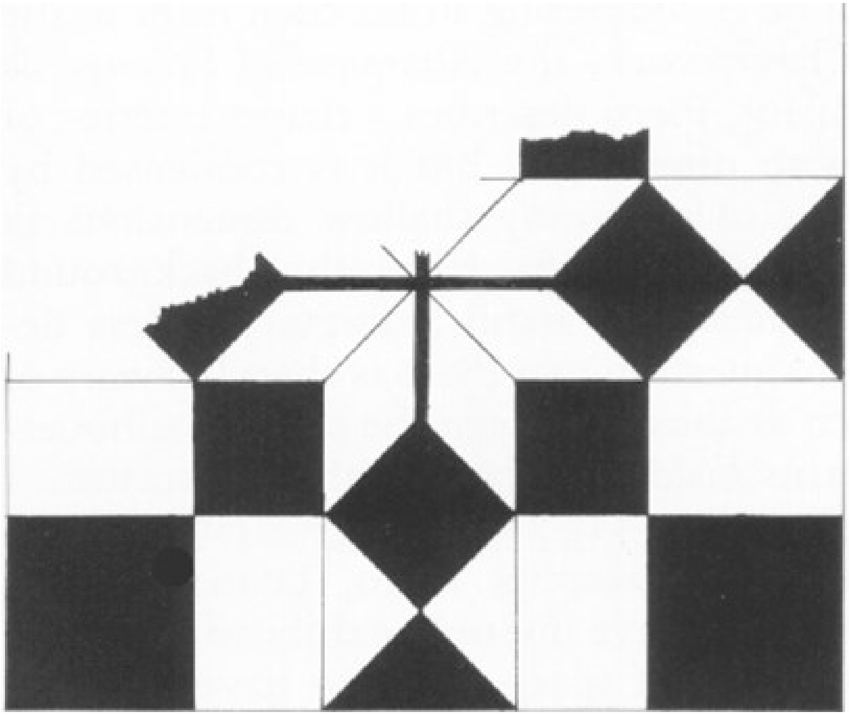
Slide from Criminisi

# Analyzing patterns and shapes

**2 patterns have been discovered!**

**Automatic rectification**



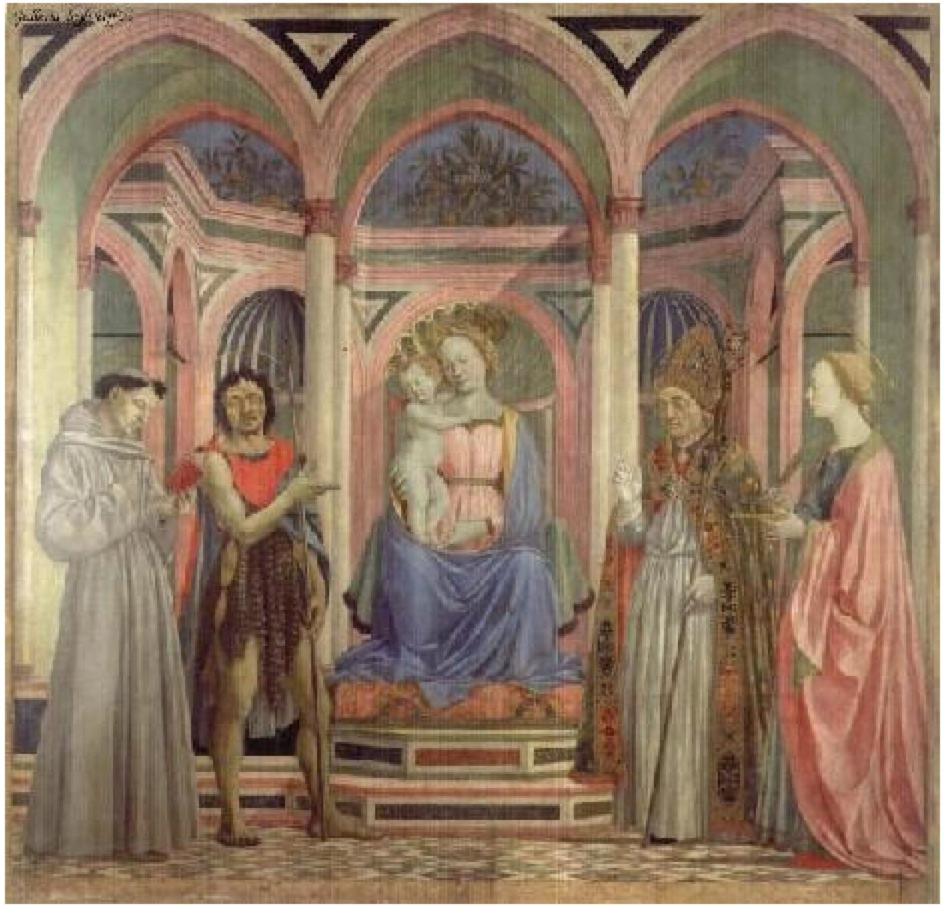


**From Martin Kemp *The Science of Art***
***(manual reconstruction)***

Slide from Criminisi

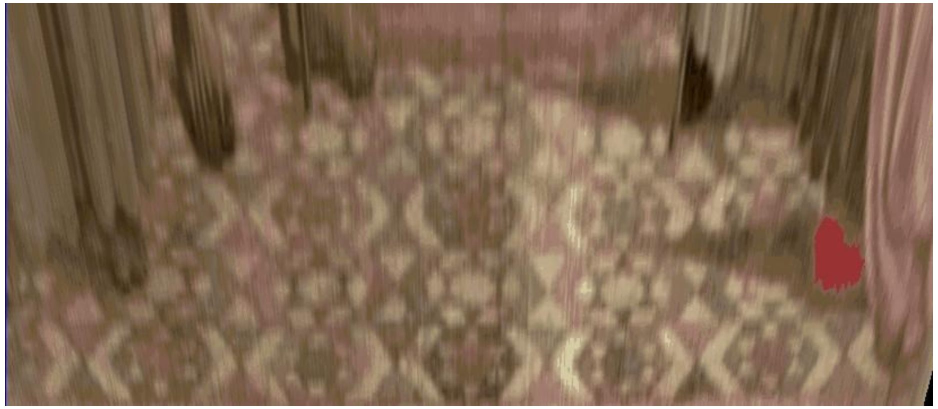
Bringing Pictorial Space to Life: Computer Techniques for the Analysis of Paintings
Antonio Criminisi , Martin Kemp, and Andrew Zisserman

# Analyzing patterns and shapes



*St. Lucy Altarpiece,* D. Veneziano

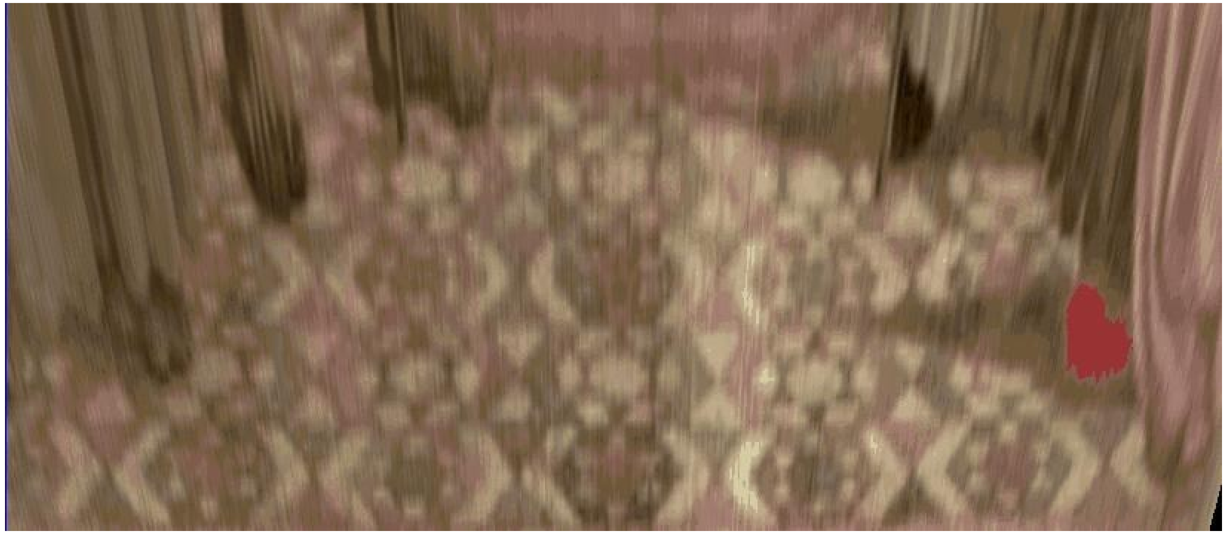
What is the (complicated) shape of the floor pattern?



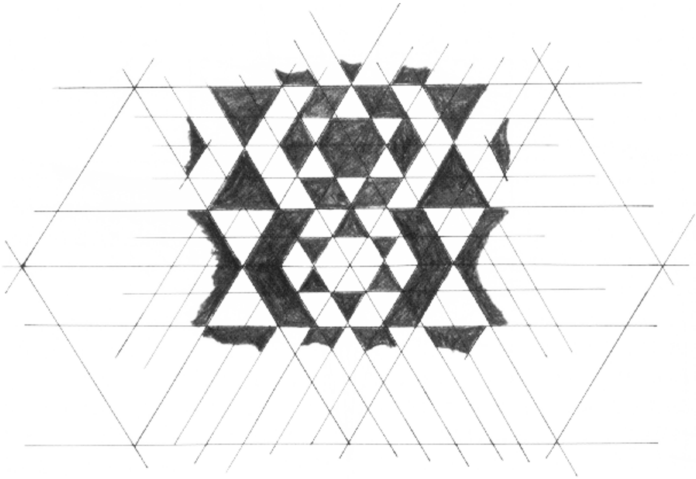
Automatically rectified floor

Slide from Criminisi

# Analyzing patterns and shapes



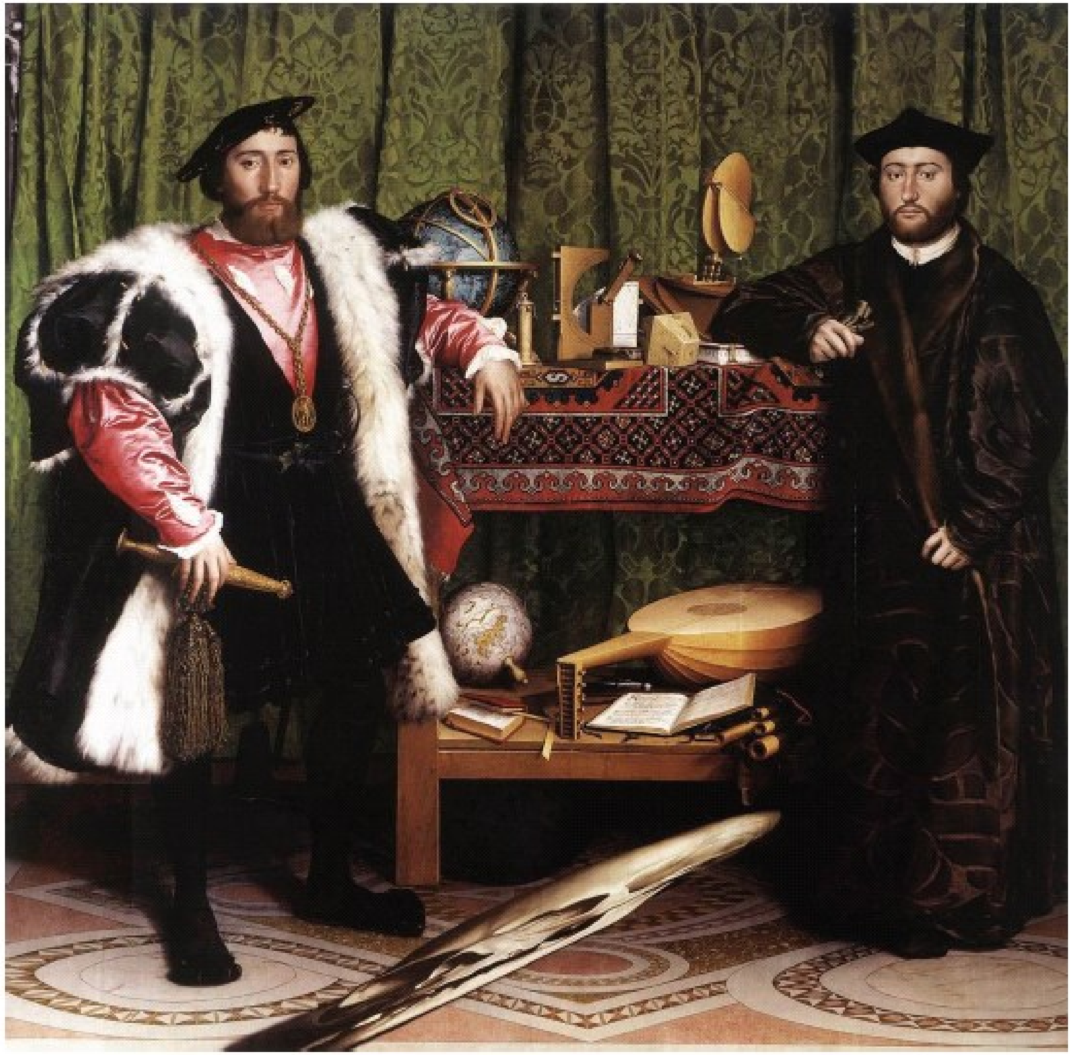
**Automatic rectification**

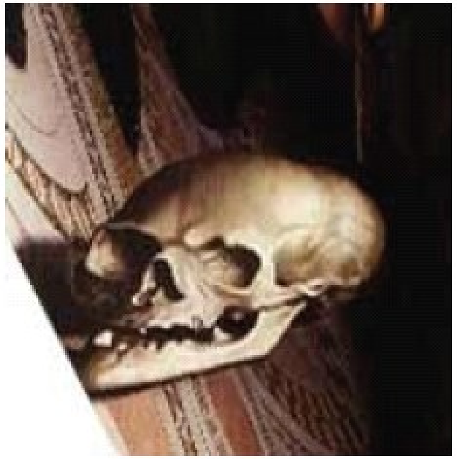


**From Martin Kemp, *The Science of Art* *(manual reconstruction)***

Slide from Criminisi

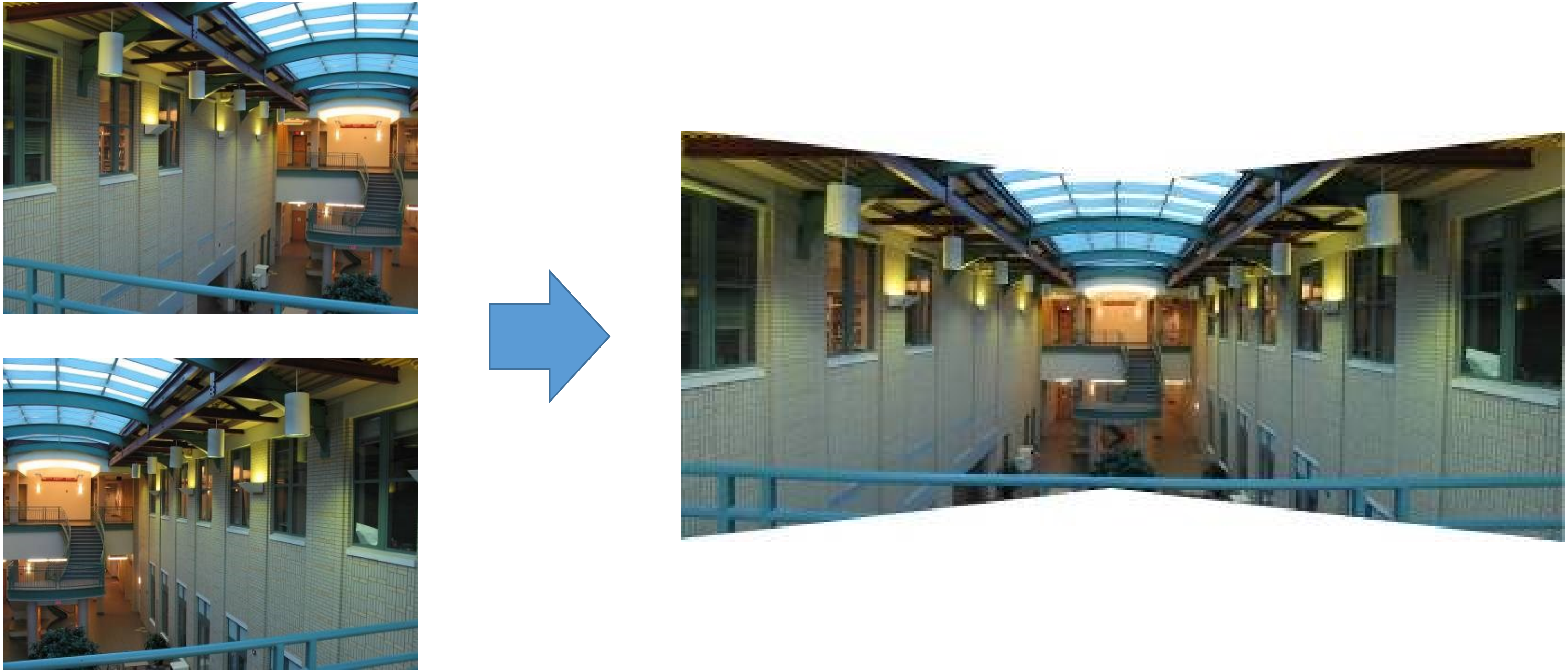
# Analyzing patterns and shapes





The Ambassadors by Hans Holbein the Younger, 1533

# É una omografia



# Omografie

Homographies …

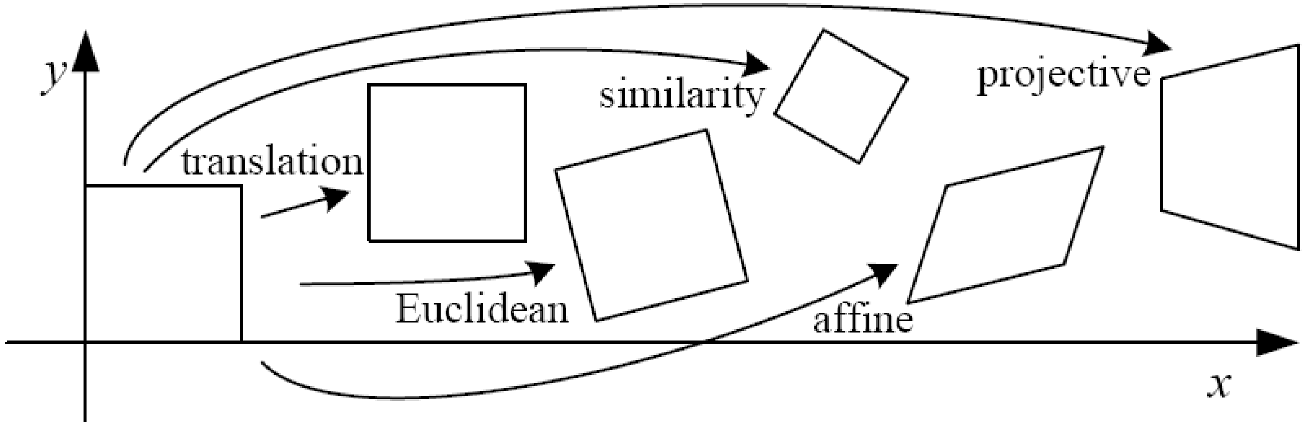
- Affine transformations, and
- Projective warps

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Properties of projective transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Closed under composition

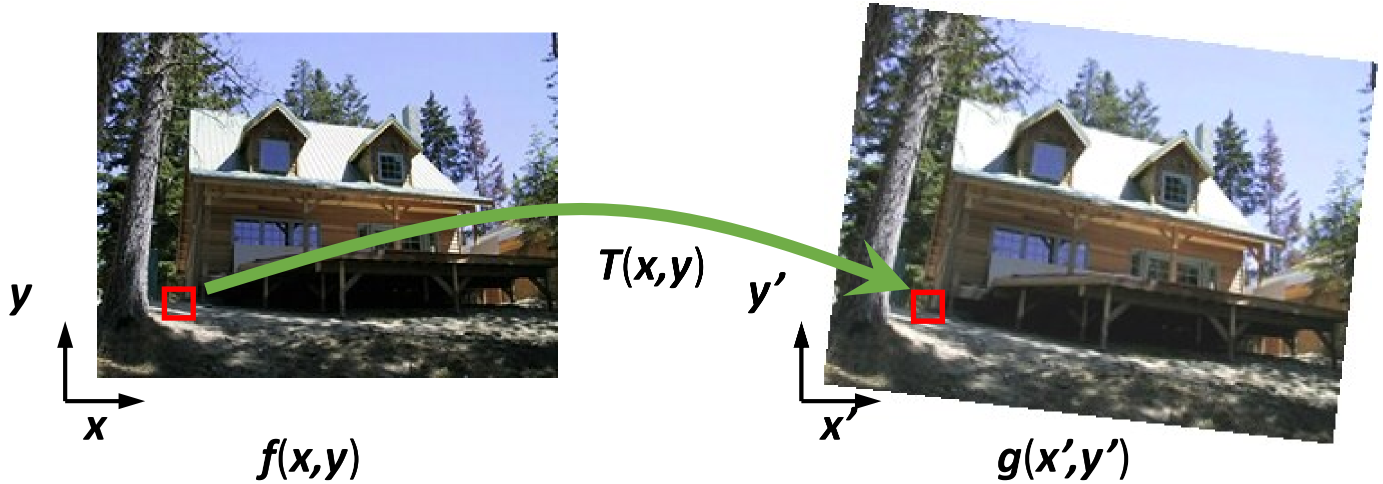
# Ricapitolando



| Name | Matrix | # D.O.F. | Preserves: | Icon |
|---|---|---|---|---|
| translation | $\left[\begin{array}{c\|c} \boldsymbol{I} & \boldsymbol{t} \end{array}\right]_{2\times 3}$ | 2 | orientation $+\cdots$ | □ |
| rigid (Euclidean) | $\left[\begin{array}{c\|c} \boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$ | 3 | lengths $+\cdots$ | ◇ |
| similarity | $\left[\begin{array}{c\|c} s\boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$ | 4 | angles $+\cdots$ | ◇ |
| affine | $\left[\begin{array}{c} \boldsymbol{A} \end{array}\right]_{2\times 3}$ | 6 | parallelism $+\cdots$ | ▱ |
| projective | $\left[\begin{array}{c} \tilde{\boldsymbol{H}} \end{array}\right]_{3\times 3}$ | 8 | straight lines | ⏢ |

# Implementing image warping

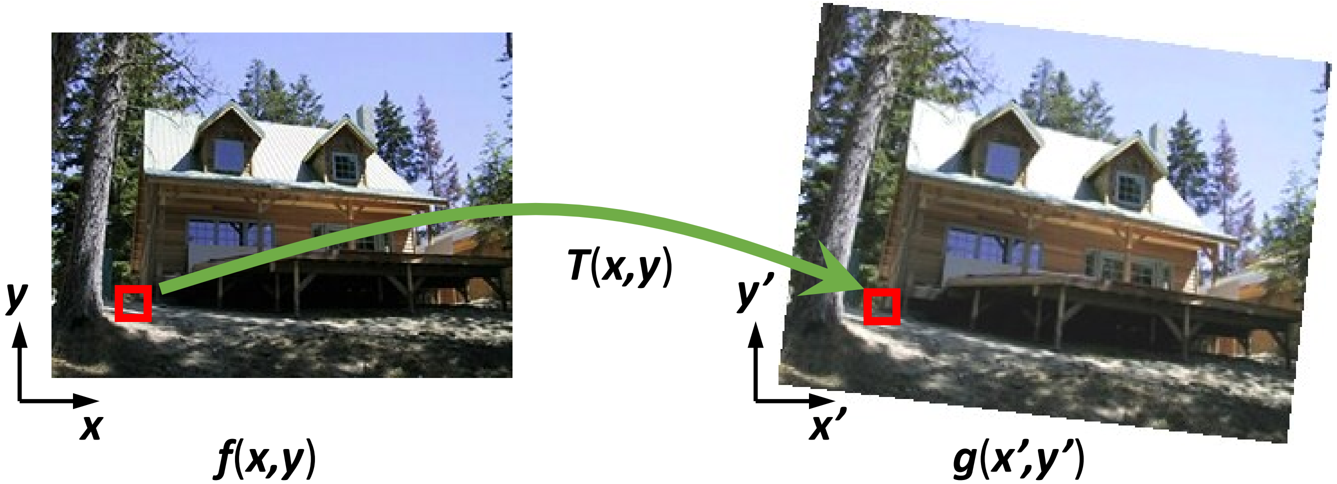
Given a coordinate transform $(\boldsymbol{x}',\boldsymbol{y}') = \boldsymbol{T}(\boldsymbol{x},\boldsymbol{y})$ and a source image $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})$, how do we compute a transformed image $\boldsymbol{g}(\boldsymbol{x}',\boldsymbol{y}') = \boldsymbol{f}(\boldsymbol{T}(\boldsymbol{x},\boldsymbol{y}))$?



# Forward Warping

Send each pixel ***f***(***x***) to its corresponding location (***x'***,***y'***) = ***T***(***x***,***y***) in ***g***(***x'***,***y'***)

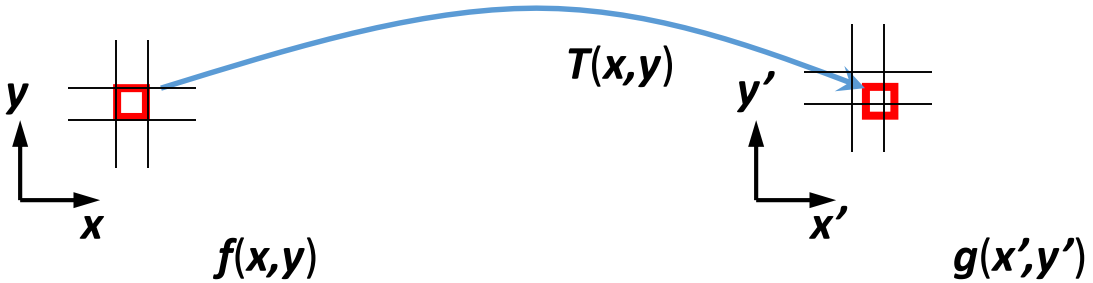
- What if pixel lands "between" two pixels?



# Forward Warping

Send each pixel ***f***(***x***) to its corresponding location (***x'***,***y'***) = ***T***(***x***,***y***) in ***g***(***x'***,***y'***)

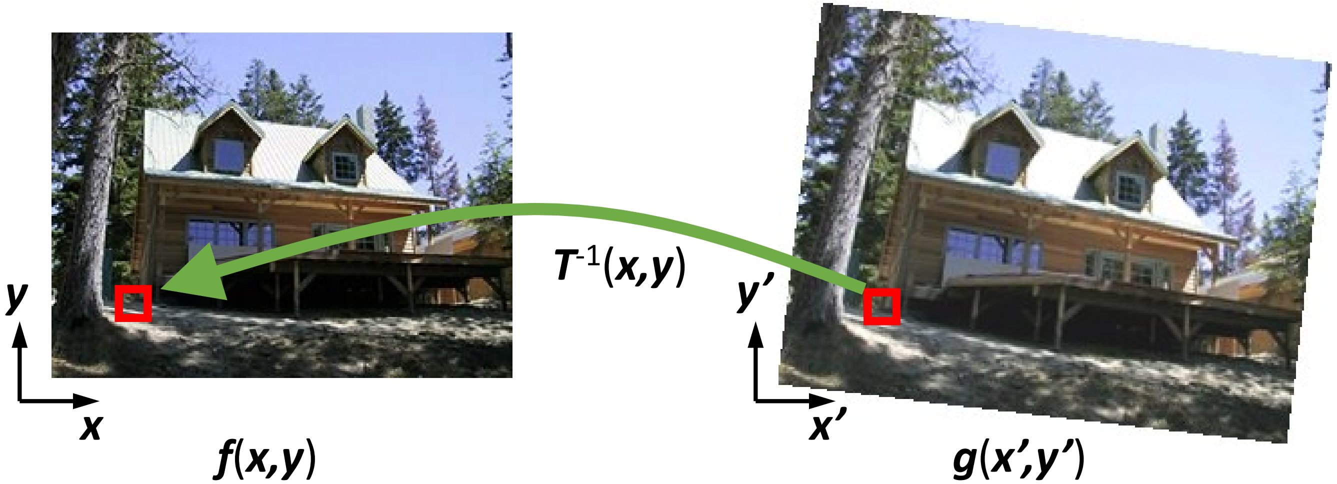
- What if pixel lands "between" two pixels?
- Answer: add "contribution" to several pixels, normalize later (splatting)
- Can still result in holes



# Inverse Warping

Get each pixel $\boldsymbol{g}(\boldsymbol{x'},\boldsymbol{y'})$ from its corresponding location $(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{T}^{-1}(\boldsymbol{x},\boldsymbol{y})$ in $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})$

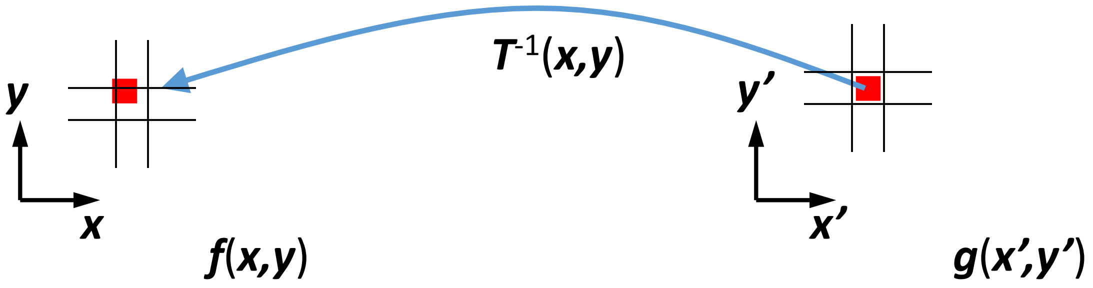
- Requires taking the inverse of the transform
- What if pixel comes from "between" two pixels?



# Inverse Warping

Get each pixel ***g***(***x'***) from its corresponding location ***x'*** = ***h***(***x***) in ***f***(***x***)

- What if pixel comes from "between" two pixels?
- Answer: *resample* color value from *interpolated* (*prefiltered*) source image



# Interpolation

Possible interpolation filters:

- nearest neighbor
- bilinear
- bicubic
- sinc

Needed to prevent “jaggies” and “texture crawl”

(with prefiltering)

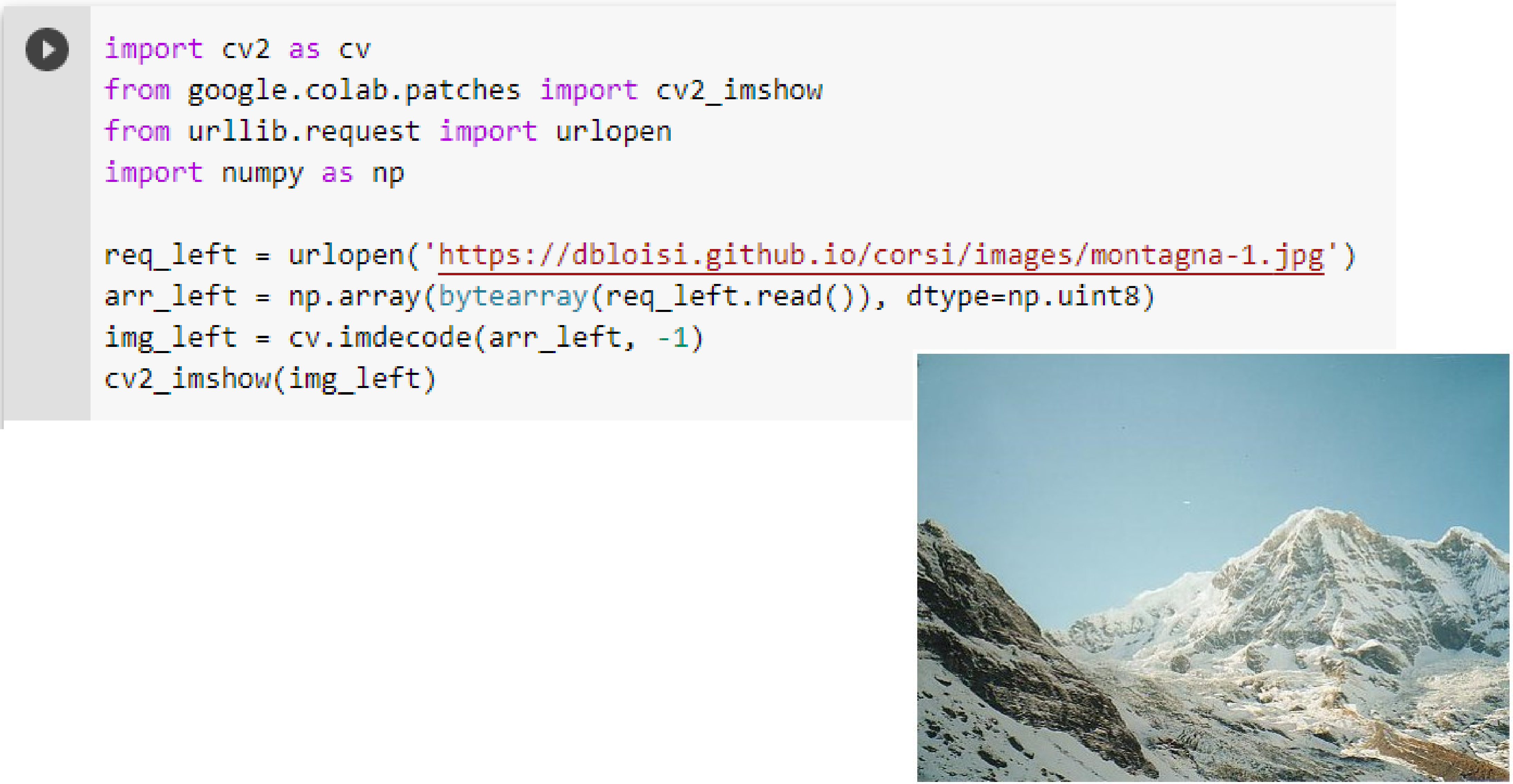
# Forward vs. inverse warping

Q: Which is better?

A: usually inverse—eliminates holes

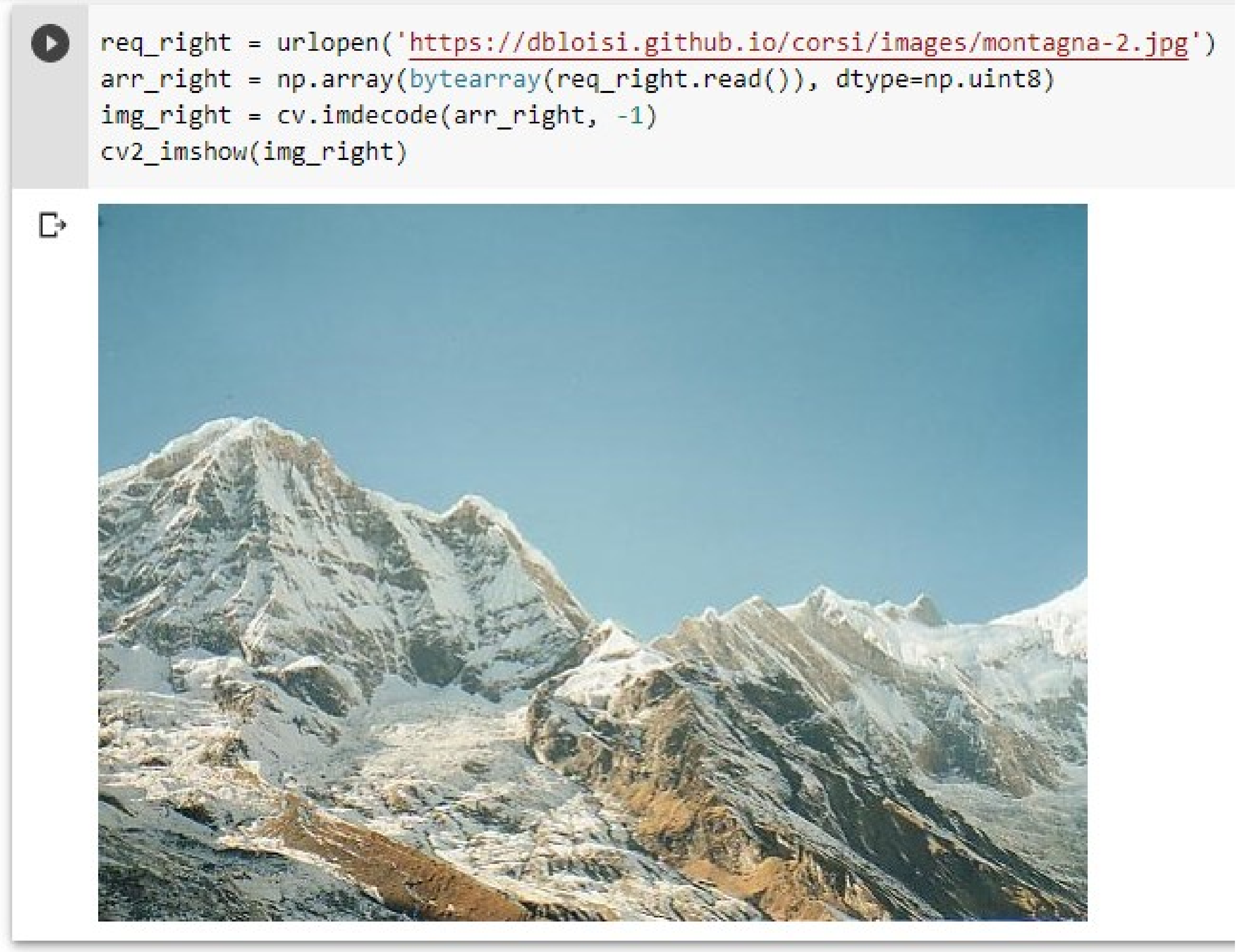
- however, it requires an invertible warp function—not always possible...

# Esempio omografia+warping



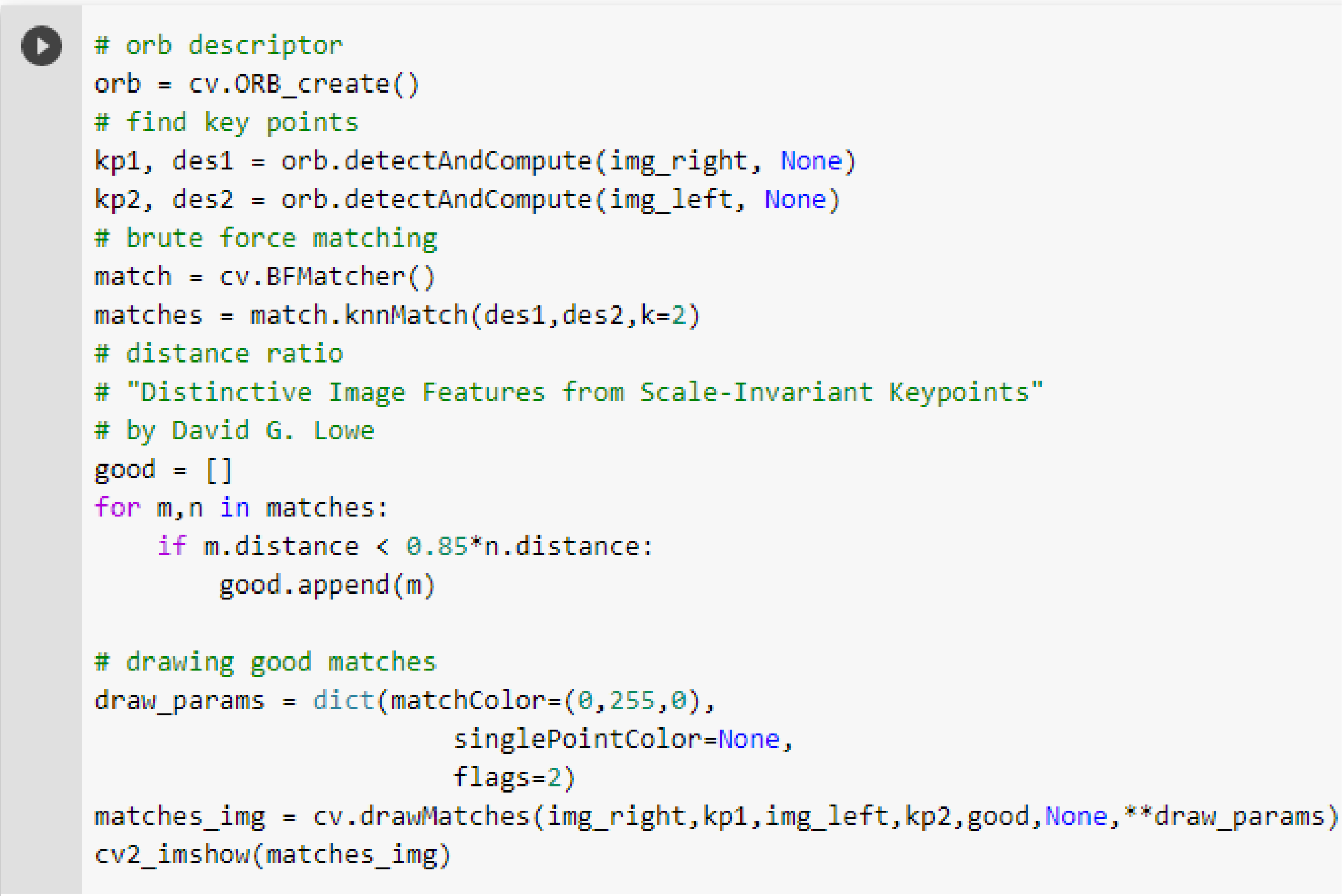
```
import cv2 as cv
from google.colab.patches import cv2_imshow
from urllib.request import urlopen
import numpy as np

req_left = urlopen('https://dbloisi.github.io/corsi/images/montagna-1.jpg')
arr_left = np.array(bytearray(req_left.read()), dtype=np.uint8)
img_left = cv.imdecode(arr_left, -1)
cv2_imshow(img_left)
```

# Esempio omografia+warping



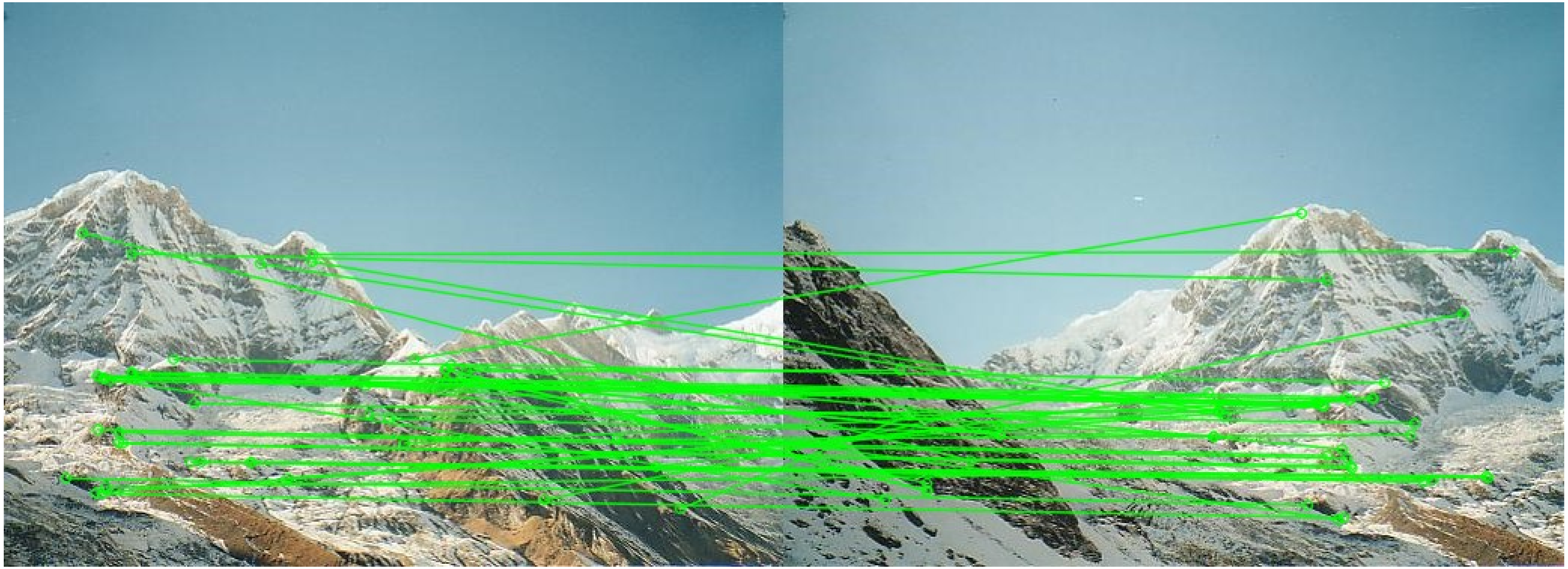
```
req_right = urlopen('https://dbloisi.github.io/corsi/images/montagna-2.jpg')
arr_right = np.array(bytearray(req_right.read()), dtype=np.uint8)
img_right = cv.imdecode(arr_right, -1)
cv2_imshow(img_right)
```

# Esempio omografia+warping

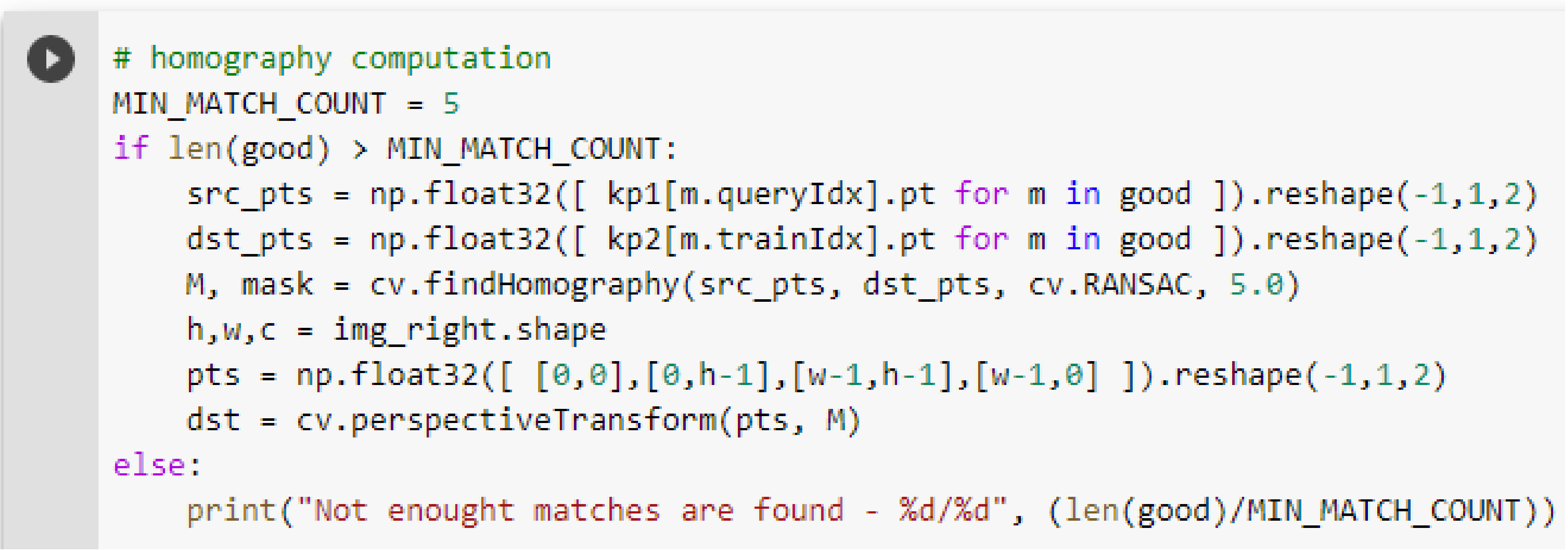


```
# orb descriptor
orb = cv.ORB_create()
# find key points
kp1, des1 = orb.detectAndCompute(img_right, None)
kp2, des2 = orb.detectAndCompute(img_left, None)
# brute force matching
match = cv.BFMatcher()
matches = match.knnMatch(des1,des2,k=2)
# distance ratio
# "Distinctive Image Features from Scale-Invariant Keypoints"
# by David G. Lowe
good = []
for m,n in matches:
    if m.distance < 0.85*n.distance:
        good.append(m)

# drawing good matches
draw_params = dict(matchColor=(0,255,0),
                       singlePointColor=None,
                       flags=2)
matches_img = cv.drawMatches(img_right,kp1,img_left,kp2,good,None,**draw_params)
cv2_imshow(matches_img)
```

# Esempio omografia+warping

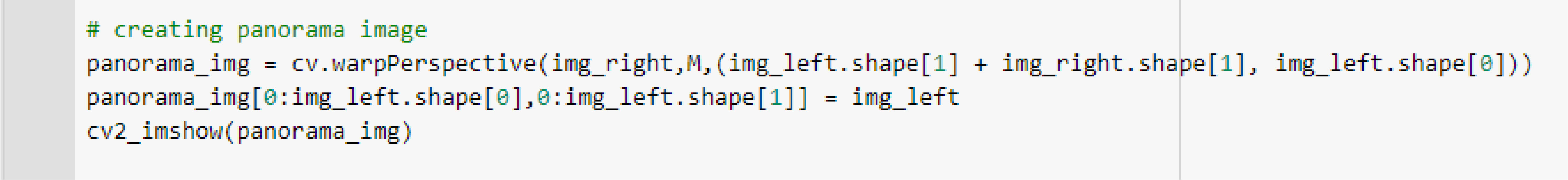


# Esempio omografia+warping

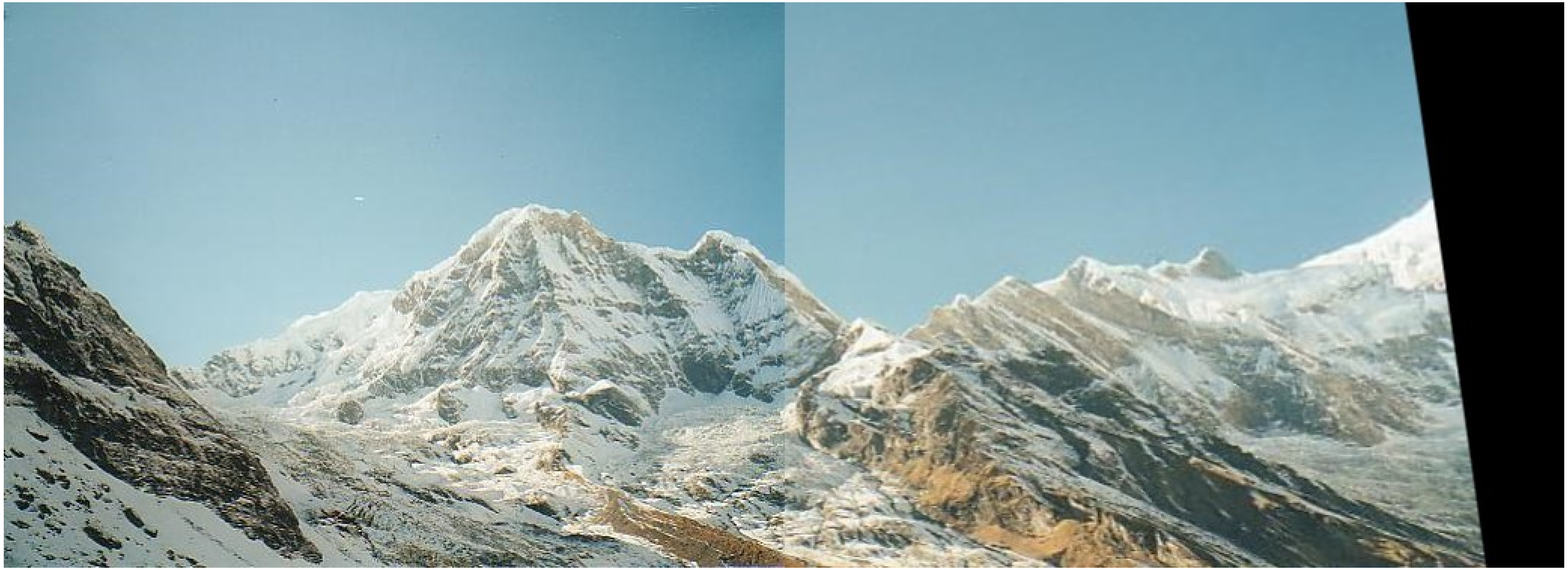


```
# homography computation
MIN_MATCH_COUNT = 5
if len(good) > MIN_MATCH_COUNT:
    src_pts = np.float32([ kp1[m.queryIdx].pt for m in good ]).reshape(-1,1,2)
    dst_pts = np.float32([ kp2[m.trainIdx].pt for m in good ]).reshape(-1,1,2)
    M, mask = cv.findHomography(src_pts, dst_pts, cv.RANSAC, 5.0)
    h,w,c = img_right.shape
    pts = np.float32([ [0,0],[0,h-1],[w-1,h-1],[w-1,0] ]).reshape(-1,1,2)
    dst = cv.perspectiveTransform(pts, M)
else:
    print("Not enought matches are found - %d/%d", (len(good)/MIN_MATCH_COUNT))
```

# Esempio omografia+warping



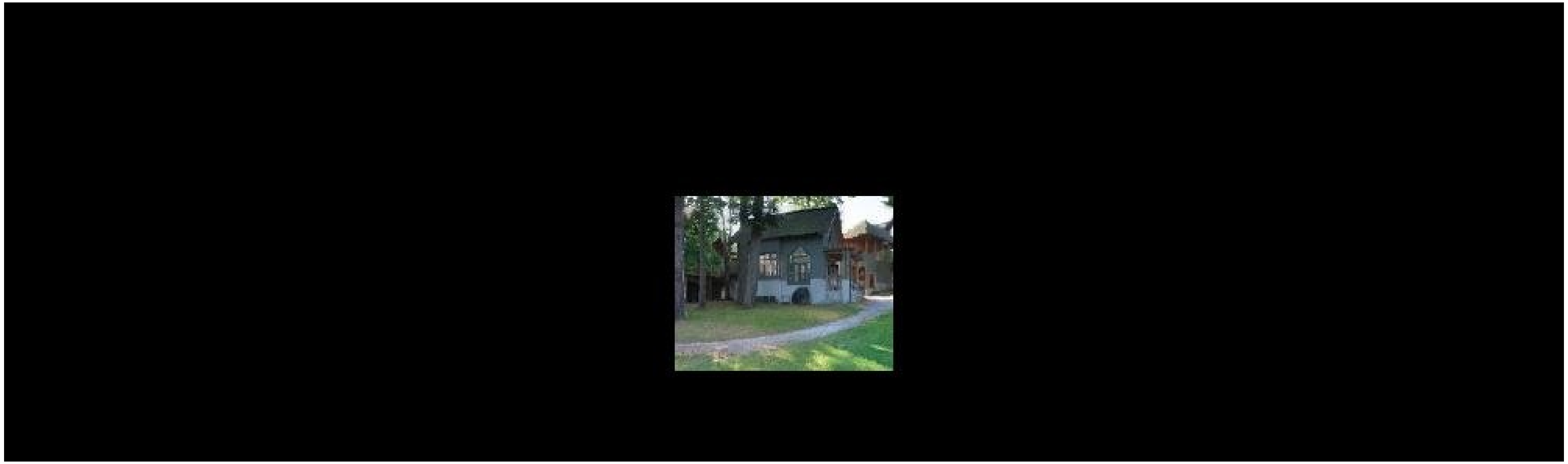
```
# creating panorama image
panorama_img = cv.warpPerspective(img_right,M,(img_left.shape[1] + img_right.shape[1], img_left.shape[0]))
panorama_img[0:img_left.shape[0],0:img_left.shape[1]] = img_left
cv2_imshow(panorama_img)
```



# Panoramas

Are you getting the whole picture?

Compact Camera FOV = 50 x 35°

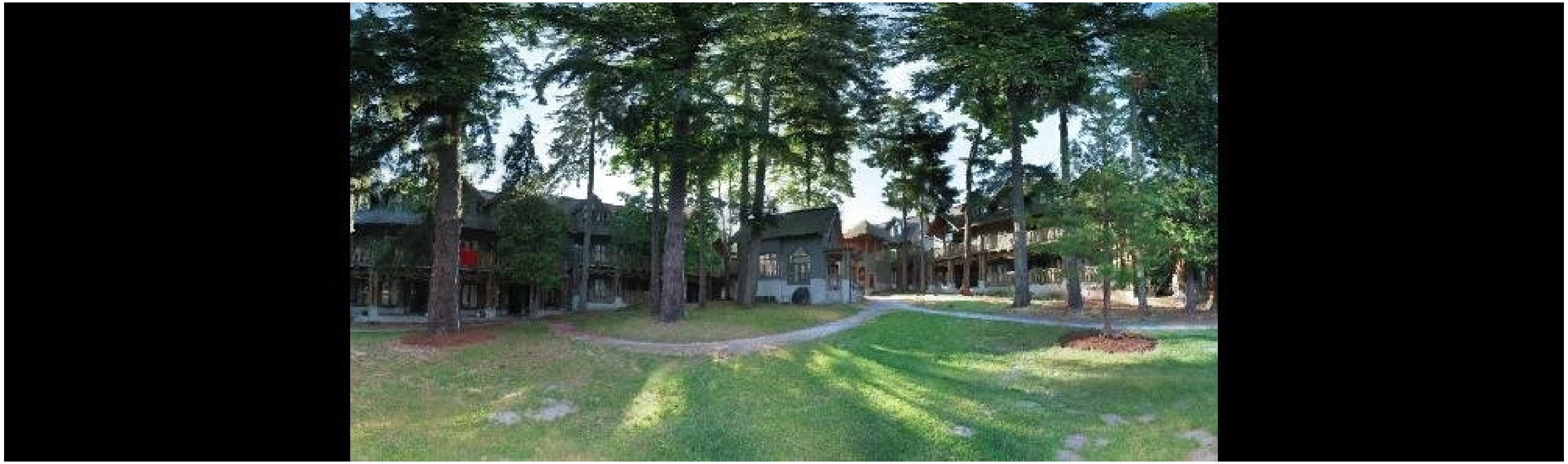


# Panoramas

Are you getting the whole picture?

Compact Camera FOV = 50 x 35°

Human FOV = 200 x 135°



# Panoramas

Are you getting the whole picture?

Compact Camera FOV = 50 x 35°

Human FOV = 200 x 135°

Panoramic Mosaic = 360 x 180°

# Why “Recognising Panoramas”?

- 1D Rotations ($\theta$)
  - Ordering $\Rightarrow$ matching images

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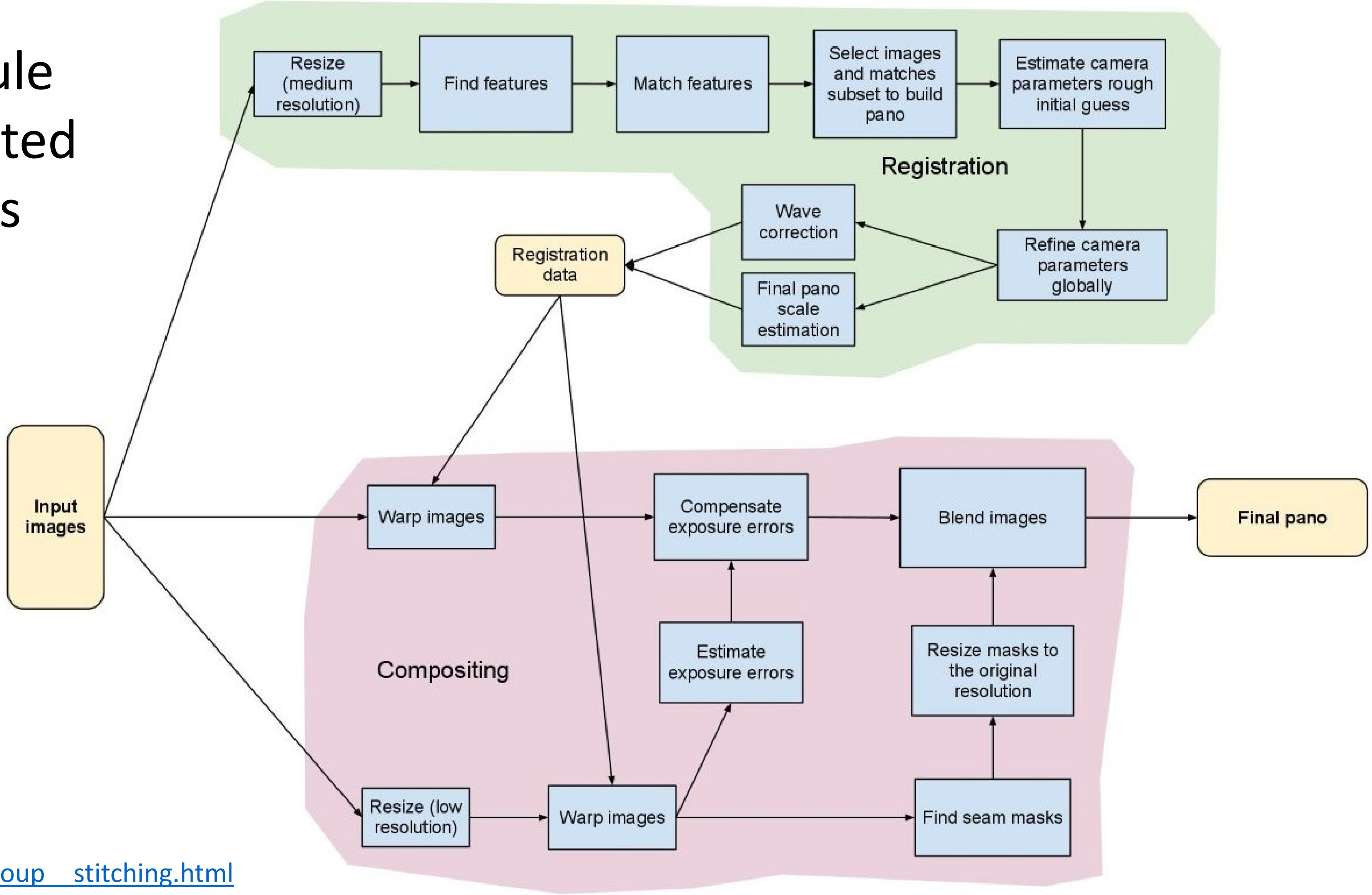
- 2D Rotations ($\theta$, $\phi$)
  - Ordering $\nRightarrow$ matching images

# Stitching in OpenCV

The stitching module pipeline implemented in the **Stitcher** class

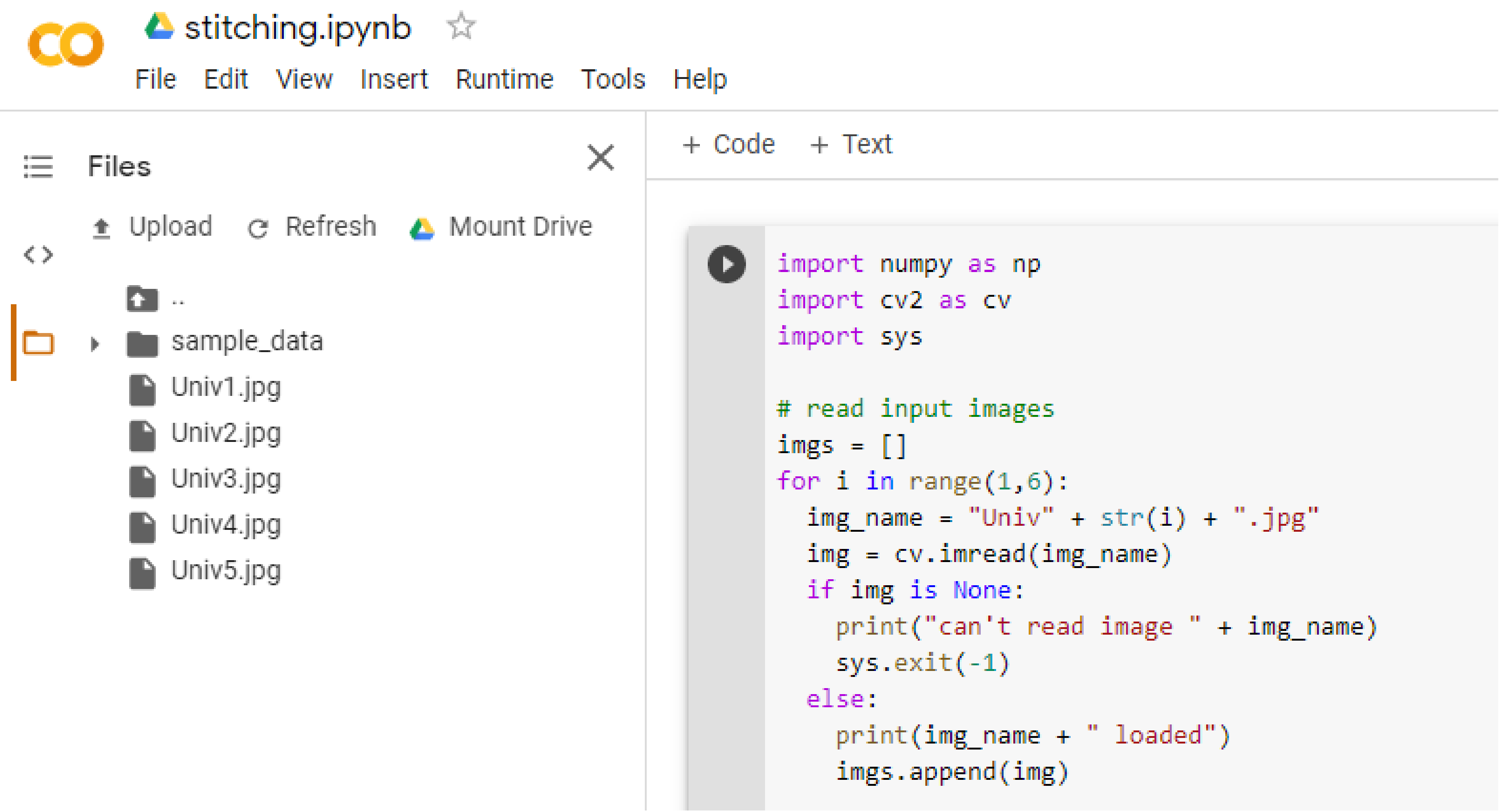


Resize (medium resolution)
Find features
Match features
Select images and matches subset to build pano
Estimate camera parameters rough initial guess
Registration
Wave correction
Registration data
Refine camera parameters globally
Final pano scale estimation
Input images
Warp images
Compensate exposure errors
Blend images
Final pano
Estimate exposure errors
Resize masks to the original resolution
Compositing
Resize (low resolution)
Warp images
Find seam masks

https://docs.opencv.org/4.1.2/d1/d46/group__stitching.html

# Stitching in OpenCV

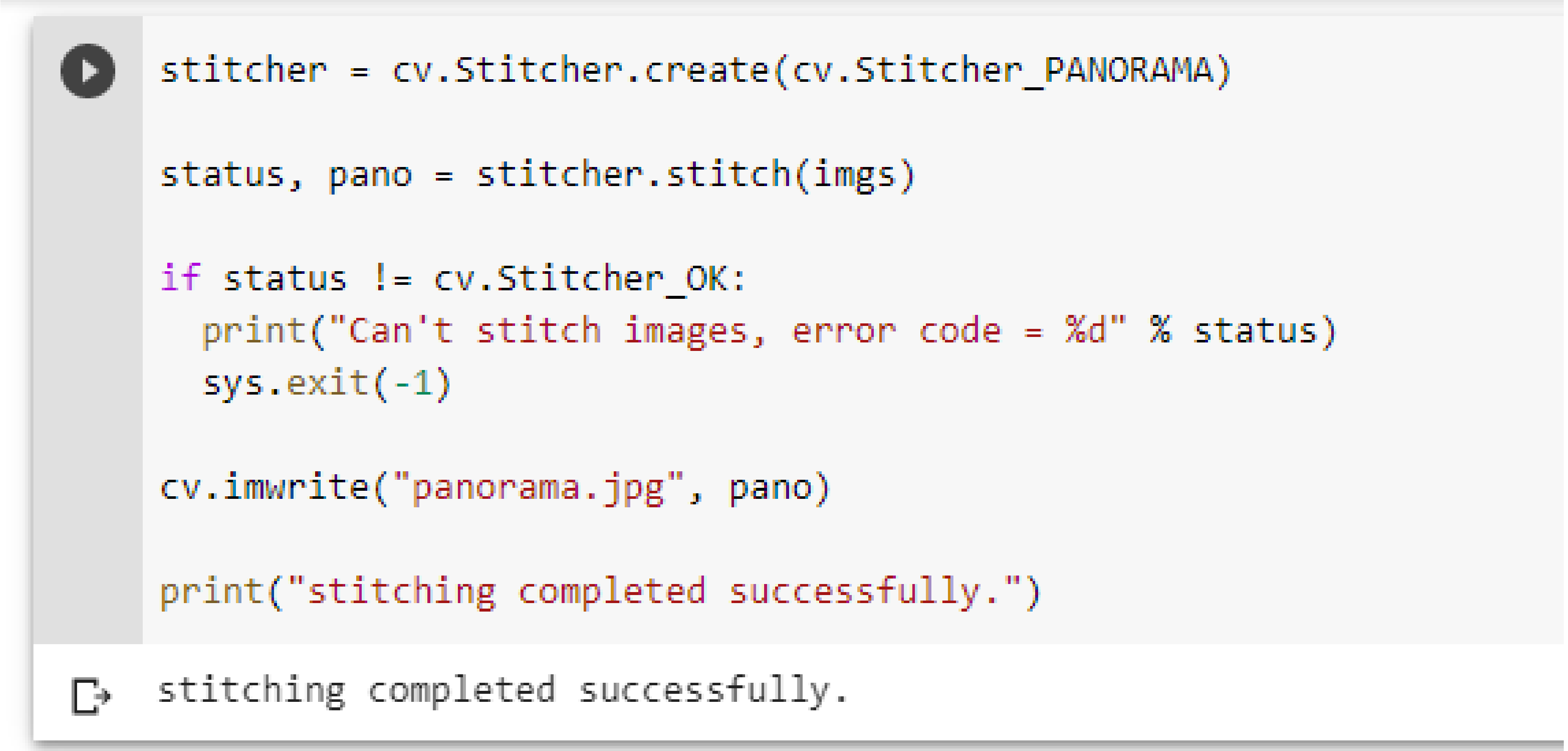
# Stitching in OpenCV



```
import numpy as np
import cv2 as cv
import sys

# read input images
imgs = []
for i in range(1,6):
  img_name = "Univ" + str(i) + ".jpg"
  img = cv.imread(img_name)
  if img is None:
    print("can't read image " + img_name)
    sys.exit(-1)
  else:
    print(img_name + " loaded")
    imgs.append(img)
```

https://docs.opencv.org/4.1.2/d5/d48/samples_2python_2stitching_8py-example.html

# Stitching in OpenCV

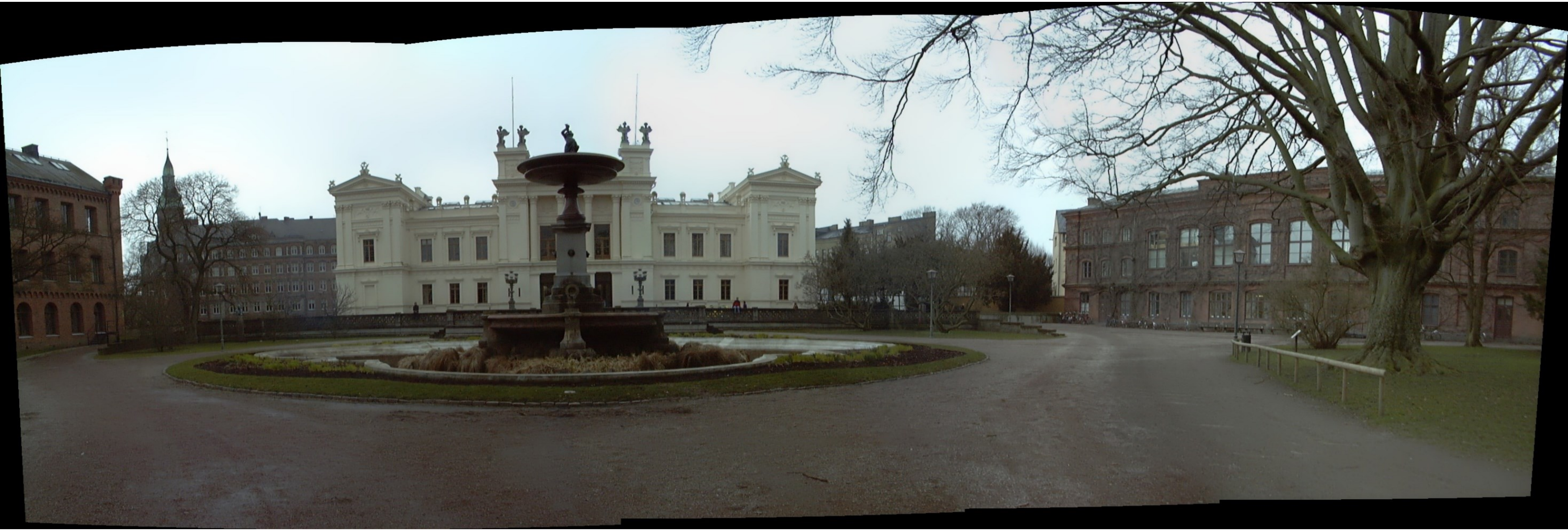


```
stitcher = cv.Stitcher.create(cv.Stitcher_PANORAMA)

status, pano = stitcher.stitch(imgs)

if status != cv.Stitcher_OK:
  print("Can't stitch images, error code = %d" % status)
  sys.exit(-1)

cv.imwrite("panorama.jpg", pano)

print("stitching completed successfully.")
```

```
stitching completed successfully.
```

https://docs.opencv.org/4.1.2/d5/d48/samples_2python_2stitching_8py-example.html

# Stitching in OpenCV



**UNIVERSITÀ DEGLI STUDI DELLA BASILICATA**

*Corso di Visione e Percezione*

Docente

Domenico D. Bloisi

# Omografie